

Derivation of the Airborne Speed Formula

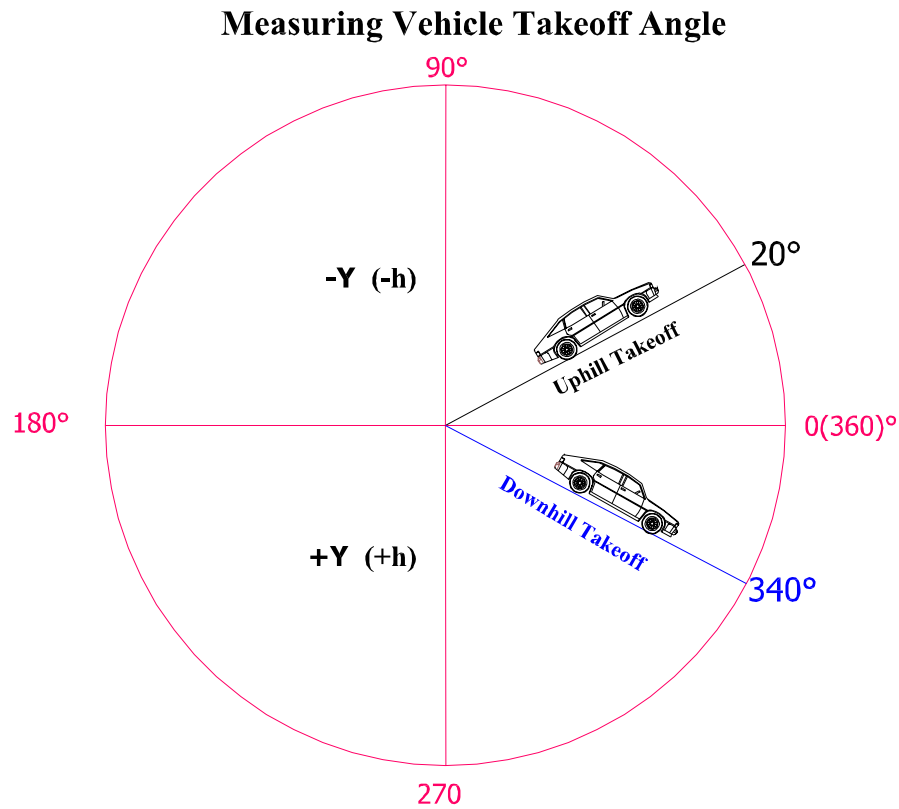
$$S = \frac{2.73 * D}{\cos\theta * \sqrt{h + D * \tan\theta}}$$



Derivation of the Airborne Speed Formula

- This Derivation results in an equation that can be used for **ALL** airborne applications
- To use the equation, you must determine three (**3**) things from the scene information
 - The horizontal distance (**D**) traveled by c/m of the vehicle from the point of takeoff to **first touch**
 - The height (**h**) that the c/m is at when the vehicle reaches the (**D**) mentioned above (**first touch**)
 - AND, the takeoff angle **θ** or the **percent of grade (m)** that exists at the point of launch

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* Use Same System (360°) as you use in Momentum Analysis



At that vault
speed, I'll
never get
them
Dukes!



Derivation of the Airborne Speed Formula

$$S = \frac{2.73 * D}{\cos\theta * \sqrt{h + D * \tan\theta}}$$

S = Speed at point of takeoff (MPH)

D = Horizontal Distance in feet traveled by Car C/M

h = Height C/M of Car from Point of Takeoff

2.73 = Mathematical Constant from Derivation

θ = Angle of Takeoff (slope) measured in degrees

Derivation of the Airborne Speed Formula

Equations are coupled w/time (Parametric)

We must look at the motion of the vehicle (speed) and first break it down into its horizontal (X) & vertical components (Y)

Lets look at the horizontal (X) direction first

Step # 1

$$X = V_o \cos \theta (T)$$

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Now once the vehicle goes airborne, for every second that it moves horizontally (**X**), it also is moving in some direction vertically (**Y**) as well

Lets now look at the vertical (**Y**) direction

+ **y** is in the same sense (direction) as gravity, **t**

Step # 2

$$y = -V_o \text{Sin } \theta(t) + \frac{1}{2} at^2$$

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X=The Horizontal Movement of the Car (Distance)

Therefore; X=D

Now *re-write* first step and substitute **D** for **X**

Step # 3

$$D = V_o \cos \theta (T)$$

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Now solve for **T** in the Equation at Step #3

Step # 4

$$T = \frac{D}{V_o \cos \theta}$$

AND

$$T^2 = \frac{D^2}{V_o^2 \cos^2 \theta}$$

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Y= The Vertical Movement of the Car (Height)

Therefore; Y=h

So substitute h for Y in Step #2 and *re-write*

Step # 5

$$h = -V_o \sin \theta(t) + \frac{1}{2} at^2$$

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Remember at Step #4 we wrote equations for **T** and **T²**

So *re-write Step #5* and substitute the equations at Step #4 for **T** and **T²**

Step # 6

$$h = -V_o \sin \theta \left(\frac{D}{V_o \cos \theta} \right) + \frac{1}{2} a \left[\frac{D^2}{V_o^2 \cos^2 \theta} \right]$$

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Take the Equation (Step #6) and replace a (acceleration) with g ;
re-write

Step # 7

$$h = -V_o \sin \theta \left(\frac{D}{V_o \cos \theta} \right) + \frac{1}{2} g \left[\frac{D^2}{V_o^2 \cos^2 \theta} \right]$$

Step # 8

Now cancel V_o in the first term:

$$h = -\sin \theta \left(\frac{D}{\cos \theta} \right) + \frac{1}{2} g \left[\frac{D^2}{V_o^2 \cos^2 \theta} \right]$$

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Now take $\frac{1}{2}$ and g in second term and multiply them both

Step # 9

$$h = -\text{Sin } \theta \left(\frac{D}{\text{Cos } \theta} \right) + \frac{g}{2} \left[\frac{D^2}{V_o^2 \text{Cos}^2 \theta} \right]$$

Step # 10

Now multiply **first two terms** on left of equal sign

$$h = -\left(\frac{D * \text{Sin } \theta}{\text{Cos } \theta} \right) + \frac{g}{2} \left[\frac{D^2}{V_o^2 \text{Cos}^2 \theta} \right]$$

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Next take the **D** and remove it from the fraction that was produced from last step

$$\frac{D * \sin \theta}{\cos \theta}$$

Rewrite:

Step # 11

$$h = -D \frac{\sin \theta}{\cos \theta} + \frac{g}{2} \left[\frac{D^2}{V_o^2 \cos^2 \theta} \right]$$

SINCE:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

Substitute terms and rewrite:

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Step # 12

$$h = -D \tan \theta + \frac{g}{2} \left[\frac{D^2}{V_o^2 \cos^2 \theta} \right]$$

Now **multiply** terms on right of plus sign and *rewrite*:

$$h = -D \tan \theta + \frac{g * D^2}{2 \left[V_o^2 \cos^2 \theta \right]}$$

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Next move the minus term across equal sign, this will change its mathematical sign to positive; *rewrite*:

Step # 13

$$h + D \tan \theta = \frac{g * D^2}{2 \left[V_o^2 \cos^2 \theta \right]}$$

REMEMBER: The purpose of this exercise is to *derive* an equation that determines *speed (or Velocity)* of a vehicle that goes airborne. So we need to **isolate that (*V*)** in the equation

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Step # 14

$$\frac{2[h + D \tan \theta] \cos^2 \theta}{g * D^2} = \frac{1}{V_o^2}$$

Next move **both terms** from Step 14 equation above **across equal sign**, this will invert each term and place **Velocity term on proper side**; *rewrite*:

Step # 15

$$V_o^2 = \frac{g * D^2}{2 \cos^2 \theta [h + D \tan \theta]}$$

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Next take the **square root of each term** in equation from Step 15. This will get rid of the squared terms, particularly, the Velocity squared on the left side of the equation.

Step # 16

$$\sqrt{V_o^2} = \frac{\sqrt{g} * \sqrt{D^2}}{\sqrt{2} \sqrt{\cos^2 \theta} \sqrt{[h + D \tan \theta]}}$$

Now, **Rewrite** the equation

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Step # 17

$$V_o = \frac{\sqrt{g * D}}{\sqrt{2} \text{Cos } \theta \sqrt{[h + D \tan \theta]}}$$

Next take the square root of g (32.2) and divide that by the square root of 2.

$$\frac{\sqrt{g}}{\sqrt{2}} = \frac{\sqrt{32.2}}{\sqrt{2}} = \frac{5.674}{1.414} = 4.01$$

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Now *rewrite* the equation and **replace the square root of g in the upper term with 4.01** (square root of 2 has been divided out)

Step # 18

$$V_o = \frac{4.01 * D}{\text{Cos } \theta \sqrt{[h + D \tan \theta]}}$$

Velocity (V_o in this case) is equal to Speed multiplied by 1.466 ($S \times 1.466$) So *rewrite* the equation and **replace V_o with $S \times 1.466$**

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Step # 19

$$S * 1.466 = \frac{4.01 * D}{\text{Cos } \theta \sqrt{[h + D \tan \theta]}}$$

Next *divide both sides* of the equal sign **by 1.466**, this will leave S (Speed in mph) **on the left side** and **2.73** will **replace 4.01** in the upper term on the right.

Now **Rewrite** the equation in its **Final Form**

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Step # 20

$$S = \frac{2.73 * D}{\cos \theta * \sqrt{h + D * \tan \theta}}$$

Summary

- This Derivation has yielded an equation that can be used for **ALL** airborne applications
- The measurements needed for using the formula (Distance and height) are critical. The accuracy of your results are also very sensitive to this fact.
- Therefore a “Rule of Thumb” for its use in the field
- **Never** Measure **D (Distance)** to long or **h (height)** to short

For Further Information & Practice

- Chapter VIII in the Textbook (FTAR)
- Readings on “*Uniform Projectile Motion*” in a College or Upper Level Physics Book
- Additional Exercises in the Textbook or re-working the Class Projects
- **Fundamentals of Applied Physics for Traffic Accident Investigators**
(Daily & Shigemura, IPTM 1997)

THE END