Accounting for Impulse and Rotation: An application to left-turn, failure to yield collisions

John G. Daily, M.S.M.E.*
Jeremy S. Daily, Ph.D., P.E.†

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*President, Jackson Hole Scientific Investigations, Inc.
†Assistant Professor in Mechanical Engineering at the University of Tulsa

Abstract

A case study in which a straight truck turned left into the path of an oncoming motorcycle, killing both the rider and passenger on the motorcycle will be examined. The speed of the truck was not high but it had quantifiable angular momentum because of its turning movement. The truck's front end was further displaced laterally by the impact of the motorcycle. Using concepts from planar impact mechanics enables us to determine the impact speed of the motorcycle. A discussion of sensitivity and uncertainty accompanies the speed analysis.

Introduction

Motivation

- The principle of the Conservation of Linear Momentum (COLM) requires external force to be small compared to collision forces.

$$m_1 v_{1,in} + m_2 v_{2,in} = m_1 v_{1,out} + m_2 v_{2,out}$$

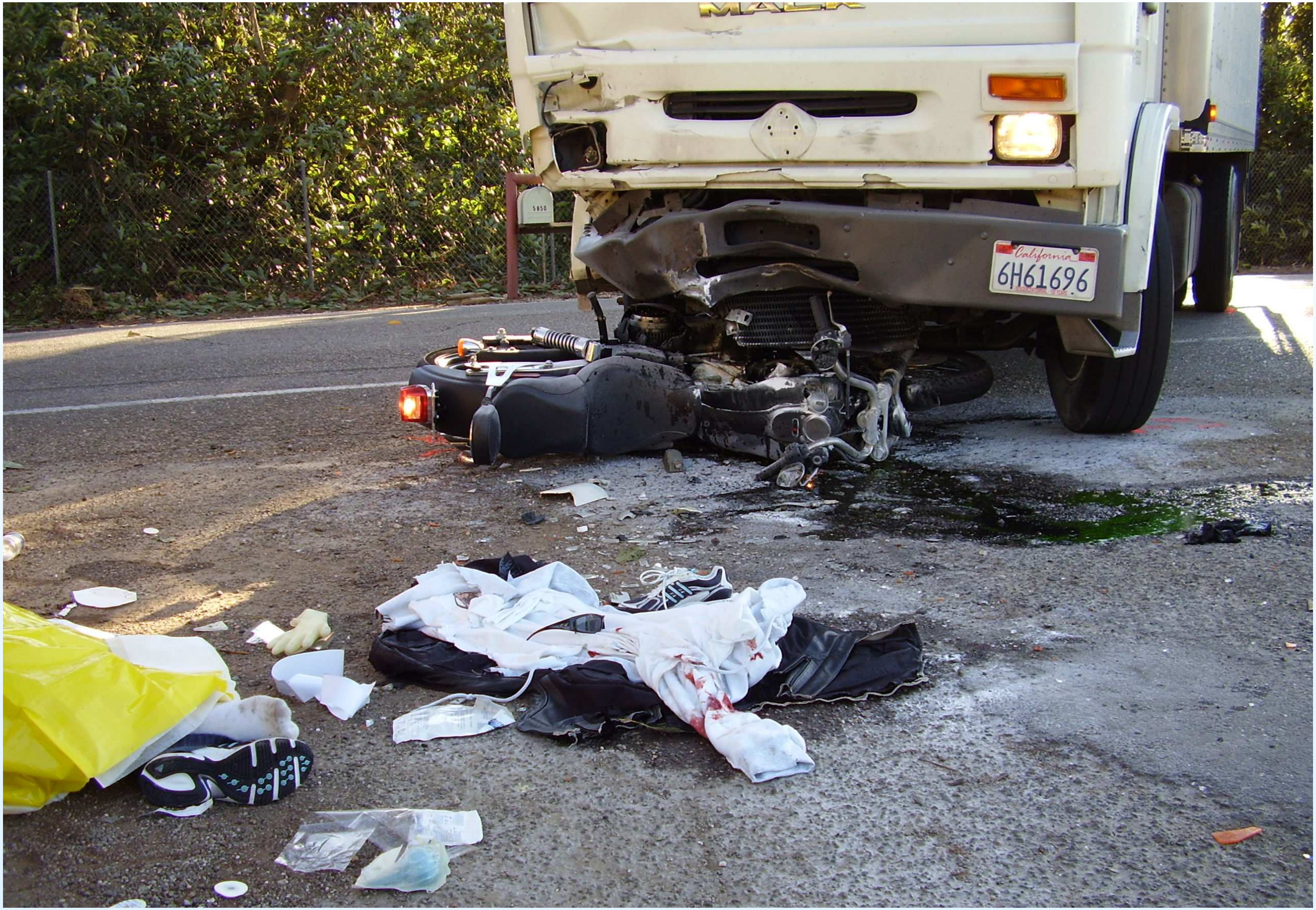
  does not include rotational aspects of planar impact mechanics.
- Including rotation and external forces from friction enable us to solve for impact speeds in non-traditional situations
- Demonstration of this procedure is given using a case study.

Case Study

Synopsis

- Mack delivery truck turns left in front of a motorcycle
- Driver claims to have never seen them
- Also claims to be stopped while in turn
- Rural highway
  - 700 ft sight distance
  - 55 mph speed limits
- Fatal for the motorcycle driver and passenger
- Original vehicles and crash site are available

Scene Photo



Skidmark Photo

• Abstract

Introduction

Case Study

• Synopsis

• Scene Photo

• Skidmark Photo

• Reconstruction Objectives

• On-site Investigation

• Positions at Time of Impact

• Determine Truck Position

Properties of the Vehicles

Motorcycle Speed Determination

Uncertainty

Discussion and Conclusions

Reconstruction Objectives

1. Determine motorcycle speed at the beginning of the skid mark.
2. Determination of the motorcycle distance from impact at certain time intervals.
3. Comment concerning the cause of the crash.

On-site Investigation Objectives

1. Determine the amount the truck was displaced from the impact of the motorcycle.
2. Measure the weights on each wheel on the Mack truck.
3. Demonstrate a likely sight picture of the truck driver and motorcycle driver at different speeds.
4. Identify the location of impact of the motorcycle with respect to the ground and with respect to the truck.
5. Measure physical truck dimensions.
6. Assess damage and deformation of the motorcycle.

On-Site Investigation

Examined both vehicles involved in the crash.

Initial evidence was still at the crash site 9 months later.

Officer who made the scene measurements assisted.

The final position of the truck could be located.

The area of impact is accurately determined due to M/C rear wheel skid mark.

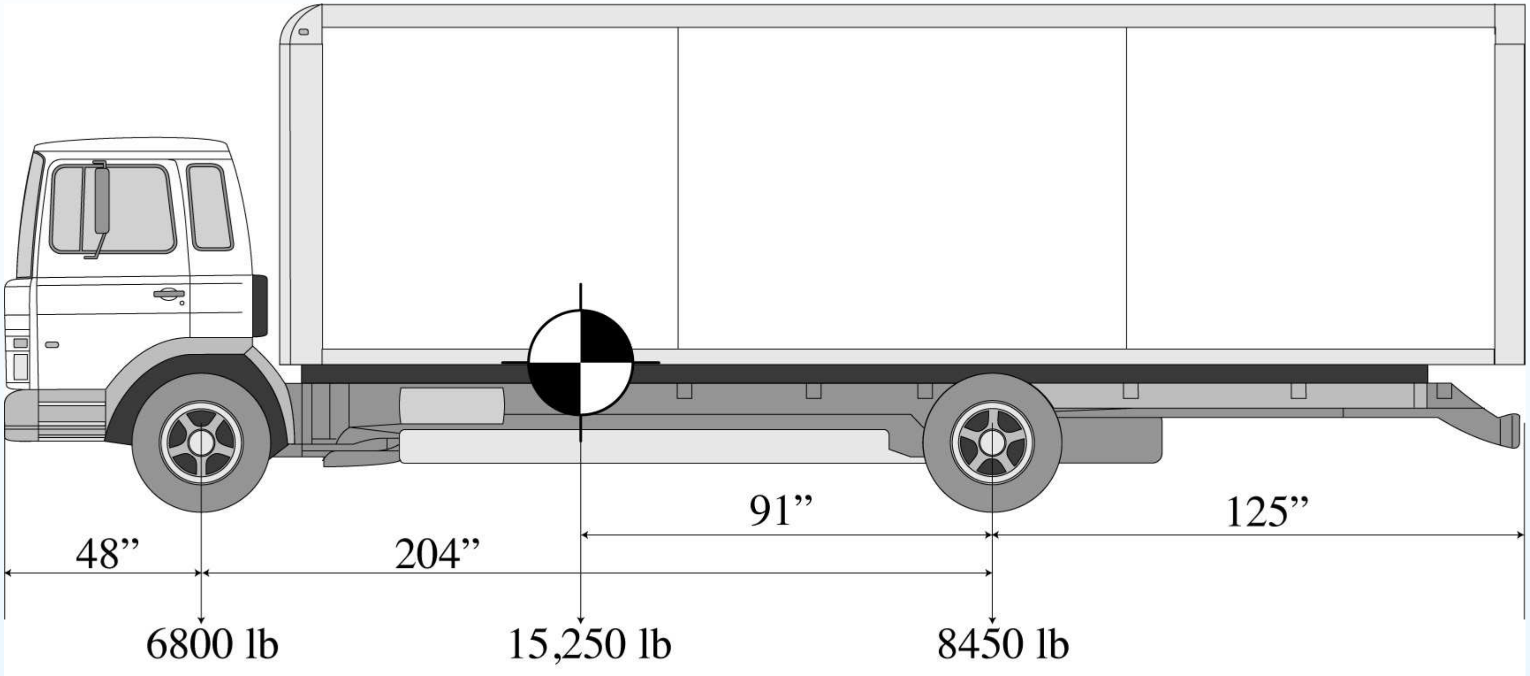
Positions at Time of Impact

Determine Truck Position

- Reverse the truck from its documented final rest position.
- Steering angle assumed constant during post impact travel.
- Investigating Officer confirmed steering angle.
- This shows the area of impact on the truck was moved 13 inches.
- The impact rotated the truck about its rear axle.

Properties of the Vehicles

Mack Truck Dimensions



$$d = wb\left(\frac{W_f}{W}\right) = 204\left(\frac{6800}{15,250}\right) = 91\,\text{in}$$

Harley Davidson Motorcycle

Weighed 667 lb and had an “always on” headlight as shown by hot shock.

Motorcycle Speed Determination

Solution Strategy

- Newton's 2nd Law of Motion:

$$impulse = [\sum F]\Delta t = m\Delta v \tag{1}$$

$$angular\,impulse = [\sum \tau_o]\Delta t = I_o\Delta\omega \tag{2}$$

  where $\sum F$ is the sum of the external forces and $\sum \tau_o$ is the sum of external torques or moments.

- Newton's 3rd Law of Motion:
  - For every action there is an equal but opposite reaction.
  - If we find the impulse on the truck, we can determine the impulse on the M/C.

Eccentric Collision Analysis

Motorcycle traveling straight at the time of impact

The lateral displacement of the area of impact was measured to be 13.5 inches.

Angle of Rotation

- Angular displacement can be determined.
- The center of rotation is the center of the rear axles.
- The angle is small and can be estimated by the arc length:

$$\theta = \frac{s}{r} = \frac{13.5}{246} = 0.05488\,\text{radians}$$

where θ is the displacement angle, s is the arc length, and r is the distance from the center of the rear axle to the area of impact.

Work-Energy Theorem for Rotation

Energy in = Energy out + work done

$$\frac{1}{2}I\omega_2^2 - \tau\theta = \frac{1}{2}I\omega_1^2 \quad (3)$$

where I is the yaw moment of inertia about the center of the rear axle,

$\omega_2 = \omega_1 + \Delta\omega$ is the post impact angular velocity of the truck,

ω_1 is the angular velocity of the truck due to turning, and

τ is the torque from friction.

Change in Angular Velocity

Rewrite Eq. (3) as:

$$\frac{1}{2}I(\omega_1 + \Delta\omega)^2 - \tau\theta = \frac{1}{2}I\omega_1^2 \tag{4}$$

where $\Delta\omega$ is the change in angular velocity due to the external impulse (collision+).

$$\Delta\omega = \sqrt{\omega_1^2 + \frac{2\tau\theta}{I}} - \omega_1 \tag{5}$$

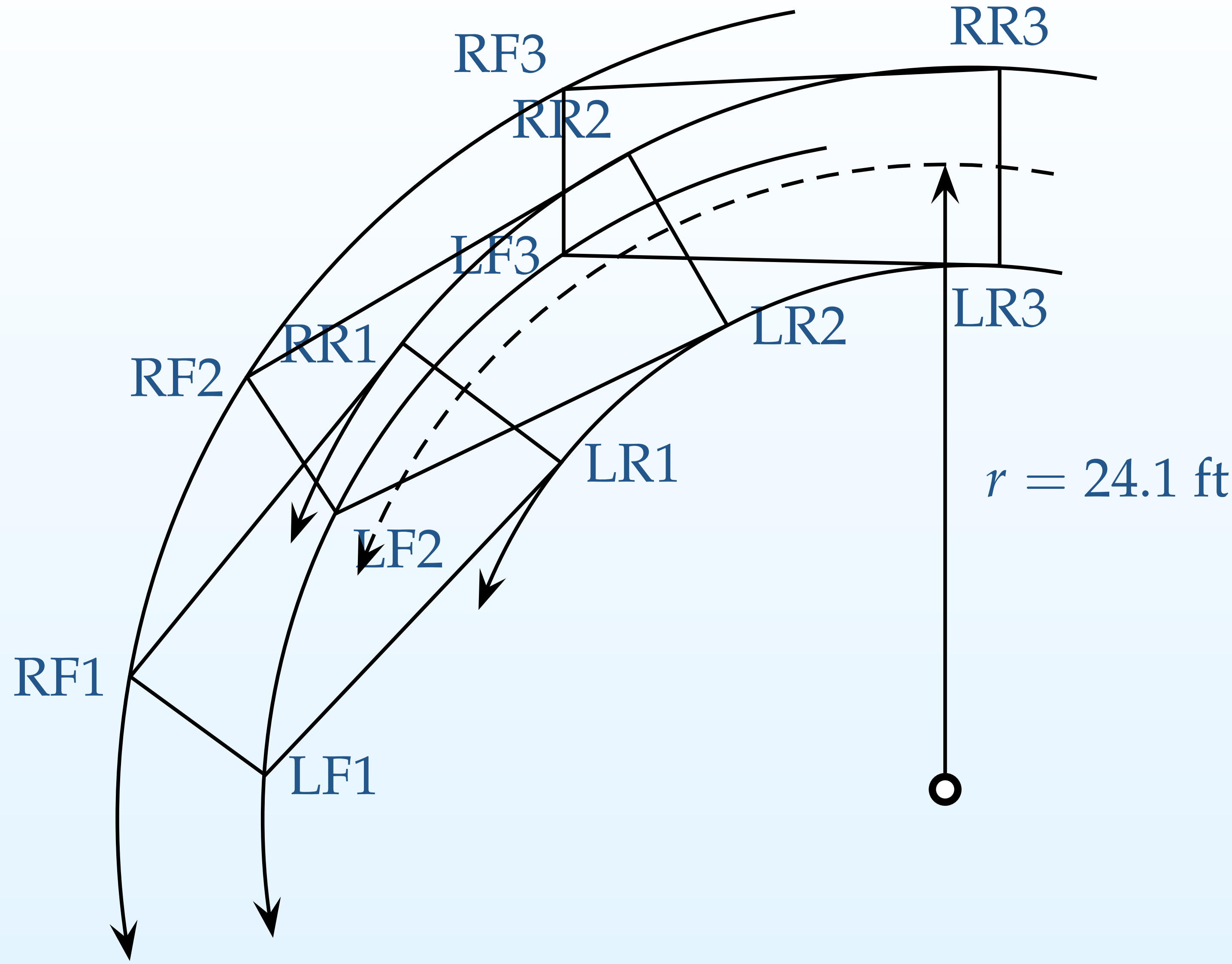
Therefore, the angular velocity of the truck in the turn, ω_1, is needed.

Determining the Turn Radius

If the path of travel of the truck is approximated by a circular arc, then three points can determine the radius of that circle.

	Right Front	Left Front	Right Rear	Left Rear
Point 1	(62.3, -4.15)	(67.5, -7.95)	(72.8, 7.4)	(78.9, 4.1)
Point 2	(66.8, 7.4)	(70.25, 2.15)	(81.5, 16.0)	(85.3, 9.4)
Point 3	(79.0, 18.5)	(79.0, 12.1)	(95.8, 19.3)	(95.8, 11.7)
Radius	31.6	26.3	27.9	20.2

Plotting the Travel Path



Turn Radius Calculation

$$r^2 = (x_1 - h)^2 + (y_1 - k)^2 \tag{6}$$

$$r^2 = (x_2 - h)^2 + (y_2 - k)^2 \tag{7}$$

$$r^2 = (x_3 - h)^2 + (y_3 - k)^2 \tag{8}$$

Subtracting Eq (6) from Eqs. (7) and (8) followed by expanding and simplifying gives:

$$x_1^2 - x_2^2 + y_1^2 - y_2^2 = (x_2 - x_1)h + (y_2 - y_1)k \tag{9}$$

$$x_1^2 - x_3^2 + y_1^2 - y_3^2 = (x_3 - x_1)h + (y_3 - y_1)k \tag{10}$$

This is a linear equation with two unknowns.

Angular velocity

- Average angular velocity in the curve is $\omega_1 = \frac{v}{r}$ where v is the forward velocity of the vehicle.
- Slide-to-stop equation $v = \sqrt{2fgd}$ where $f = 0.5$ is the drag factor of a truck with motorcycle wedged under the right front tire.
- The truck traveled about $d = 10$ ft after the collision, so

$$\omega_1 = \frac{\sqrt{2fgd}}{r} = \frac{\sqrt{2(0.5)(32.2)(10)}}{24.1} = 0.7446\,\text{rad/s}$$

Torque from Friction

The torque applied to the truck to resist rotation about the center of the rear axles comes from the lateral friction from the front tires.

$$\tau = wb \times F_{friction} \tag{11}$$

where the wheelbase $wb = 204$ inches.

$$F_{friction} = fW_f = 0.6(7050) = 4230\,\text{lb} \tag{12}$$

where $f = 0.6$ is an acceptable drag factor for a commercial vehicle tire. This means the applied torque on the truck after impact is

$$\tau = 204(4230) = 862,920\,\text{in-lb} \tag{13}$$

Yaw Moment of Inertia of Mack Truck

- University of Michigan Transportation Research Institute reports the yaw moment of inertia is nearly 177,000 in-lb-sec^2 about the COM.
- Parallel axis theorem:

$$I = I_G + md^2 \tag{14}$$

$$I = 177,000 + \left(\frac{15,250}{386.4}\right)(91)^2 = 503,830\,\text{in-lb-sec}^2 \tag{15}$$

Change in Angular Velocity

With all values determined, the change in angular velocity as a result of the impact can be determined for the truck using Eq. (5):

$$\Delta\omega = \sqrt{\omega_1^2 + \frac{2\tau\theta}{I}} - \omega_1 \quad (16)$$

$$= \sqrt{0.7446^2 + \frac{2(862,920)(0.05488)}{503,830}} - 0.7446 \quad (17)$$

$$= 0.1170\,\text{rad/s} \quad (18)$$

The change in angular velocity comes from an externally applied angular impulse.

Angular Impulse and Momentum

Fundamental text in dynamics [2] says:

$$\underbrace{I\omega_1}_{Momentum\ In} + \underbrace{\sum \tau \Delta t}_{External\ Impulse} = \underbrace{I(\omega_1 + \Delta\omega)}_{Momentum\ Out} \tag{19}$$

The external impulse on the truck comes from the collision force of the motorcycle (Newton's Third Law) and the friction of the front tires on the ground.

Linear Impulse to Angular Impulse

Radius vector is cross multiplied by the impulse vector. Therefore, the external impulse on the truck is:

$$\sum \tau \, \Delta t = r_{hit} \times m\Delta v - \tau\Delta t \tag{20}$$

The magnitude of the cross product is:

$$|r_{hit} \times m\Delta v| = m|r_{hit}||\Delta v| \sin\theta \tag{21}$$

Motorcycle Velocity Change

Eqs. (19)-(21) give an expression for the Δv of the motorcycle:

$$\Delta v = \frac{I\Delta\omega + \tau\Delta t}{m r_{hit} \sin\theta} \tag{22}$$

Δt is the duration of the crash (approx 0.150 seconds)
m is the mass of the motorcycle and riders,
r_{hit} is the distance from the rear axle to the center of pressure of the impact area (246 inches),
θ is the angle between the Δv vector and the center line of the truck.

Mass of Motorcycle

The weight of the motorcycle and rider combination is the sum of the the motorcycle weight, the driver's weight, and the passenger's weight. Therefore, the total mass is determined as

$$m = \frac{w_{motorcycle} + w_{driver} + w_{passenger}}{g} \quad (23)$$

$$= \frac{667 + 195 + 240}{386.4\,\text{in/s}^2} \quad (24)$$

$$= 2.8520\,\text{lb-sec}^2/\text{in} \quad (25)$$

The angleθ between the radius and the Δv vector must be estimated.

Impact Speed

The calculated impact speed for the motorcycle is

$$\Delta v = \frac{I\Delta\omega + \tau\Delta t}{m r_{hit} \sin\theta} \tag{26}$$

$$= \frac{(503,830)(0.1170) + 862,920(0.150)}{(2.8520)(246)\sin(30)} \tag{27}$$

$$= 546.03\,\text{in/sec} \tag{28}$$

$$= 45.50\,\text{ft/sec} \tag{29}$$

$$= 31.03\,\text{mph} \tag{30}$$

Speed at the Beginning of the Skid

Combining this impact speed with the speed loss from skidding (using a 0.7 for the drag factor), the speed the motorcycle was going at the time of the impending skid is

$$S = \sqrt{30df + S_{impact}^2} \tag{31}$$

$$= \sqrt{30(40)(0.7) + 31.03^2} \tag{32}$$

$$= \boxed{42.46\,\text{mph}} \tag{33}$$

Sensitivity and Uncertainty Analysis

Types of Uncertainty

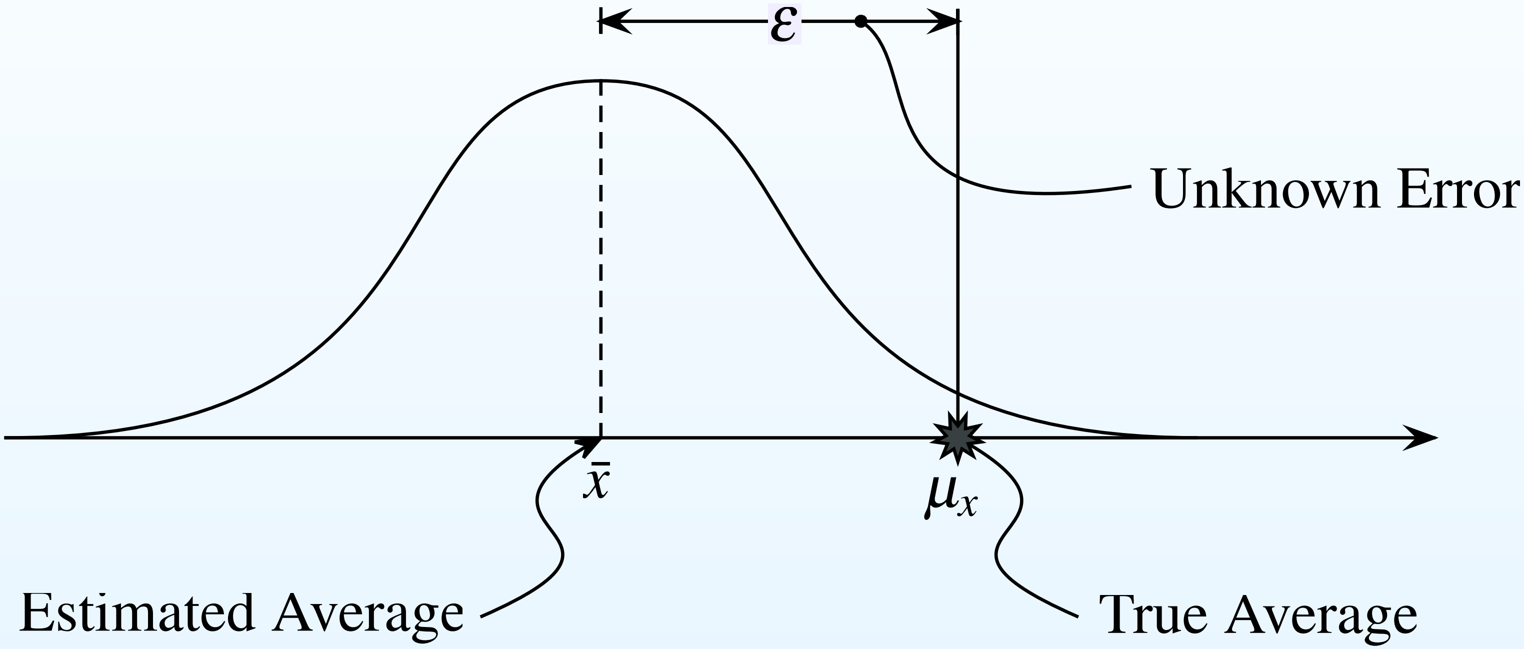
1. Aleatory (denoted with a P)
   Uncertainty from inherent randomness of phenomena. It can be quantified using repeated measurements and statistics. Synonyms include:
   (a) Random uncertainty
   (b) Precision uncertainty
2. Epistemic (denoted with a B)
   (a) Systematic Uncertainty
   (b) Bias Uncertainty

Types of Uncertainty Analysis

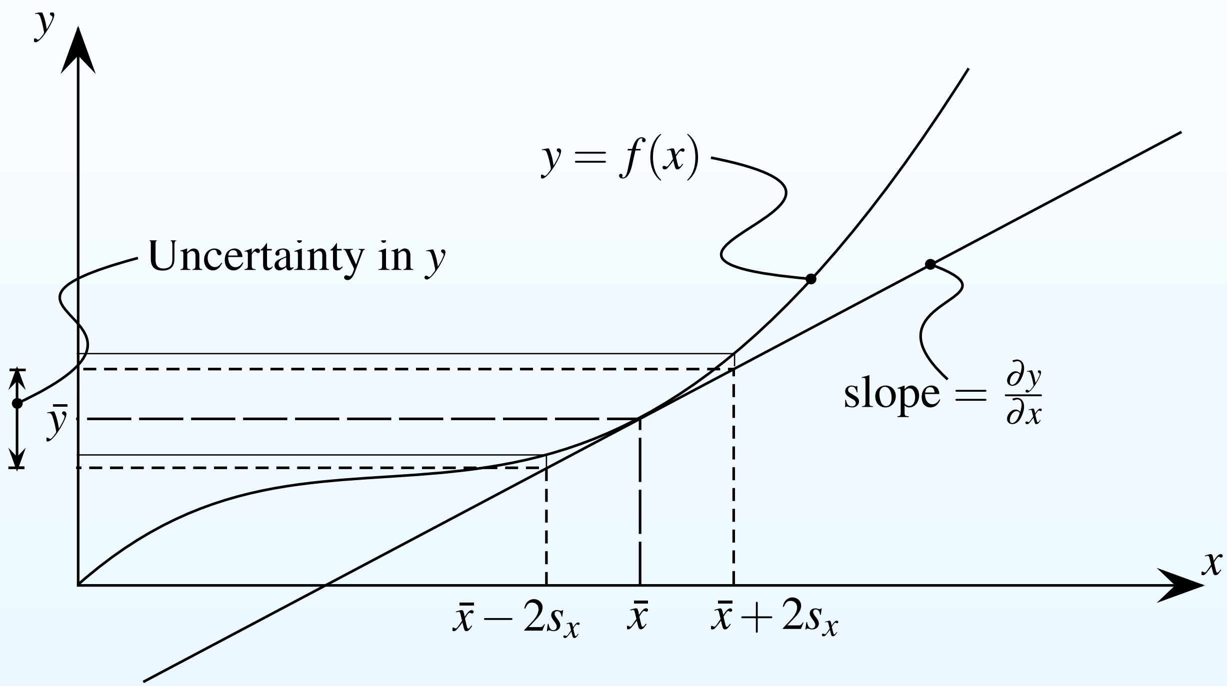
1. Differential Methods
2. Monte Carlo Sampling Methods
3. Response Surface Techniques
4. Fourier Amplitude Sensitivity

Uncertainty and Error

Uncertainty analysis is the process of giving bounds to the unknown measurement error.



Propagating Uncertainty



s_x is the standard deviation of x and $\bar{(\cdot)}$ represents the mean. The dashed line is from a differential analysis, the thin line is from a Monte Carlo Analysis.

Differential Methods

Given a function $y = f(x_1, x_2, \cdots x_n)$, we can estimate the uncertainty in y if we know the uncertainties in x.

1. Compute sensitivity gradients analytically $\frac{\partial y}{\partial x_i}$ or using finite differences $\frac{y(x)-y(x+\Delta x)}{\Delta x}$
2. Combine elemental uncertainty as:

$$u_y = \sqrt{\sum_{i=1}^{n} \left\{ \left(\frac{\partial y}{\partial x_i} B_i \right)^2 + \left(t_{\nu,95\%} \frac{\partial y}{\partial x_i} P_i \right)^2 \right\}} \qquad (34)$$

where B_i are the "bias" uncertainties and P_i are the "precision" uncertainties.

Monte Carlo Techniques

Basic Procedure

1. Assign each input variable a distribution
2. Sample from each distribution at random and combine to form a result
3. Repeat many times to generate a result distribution
4. Construct uncertainty intervals based on final distribution

Monte Carlo Example

Input variables:

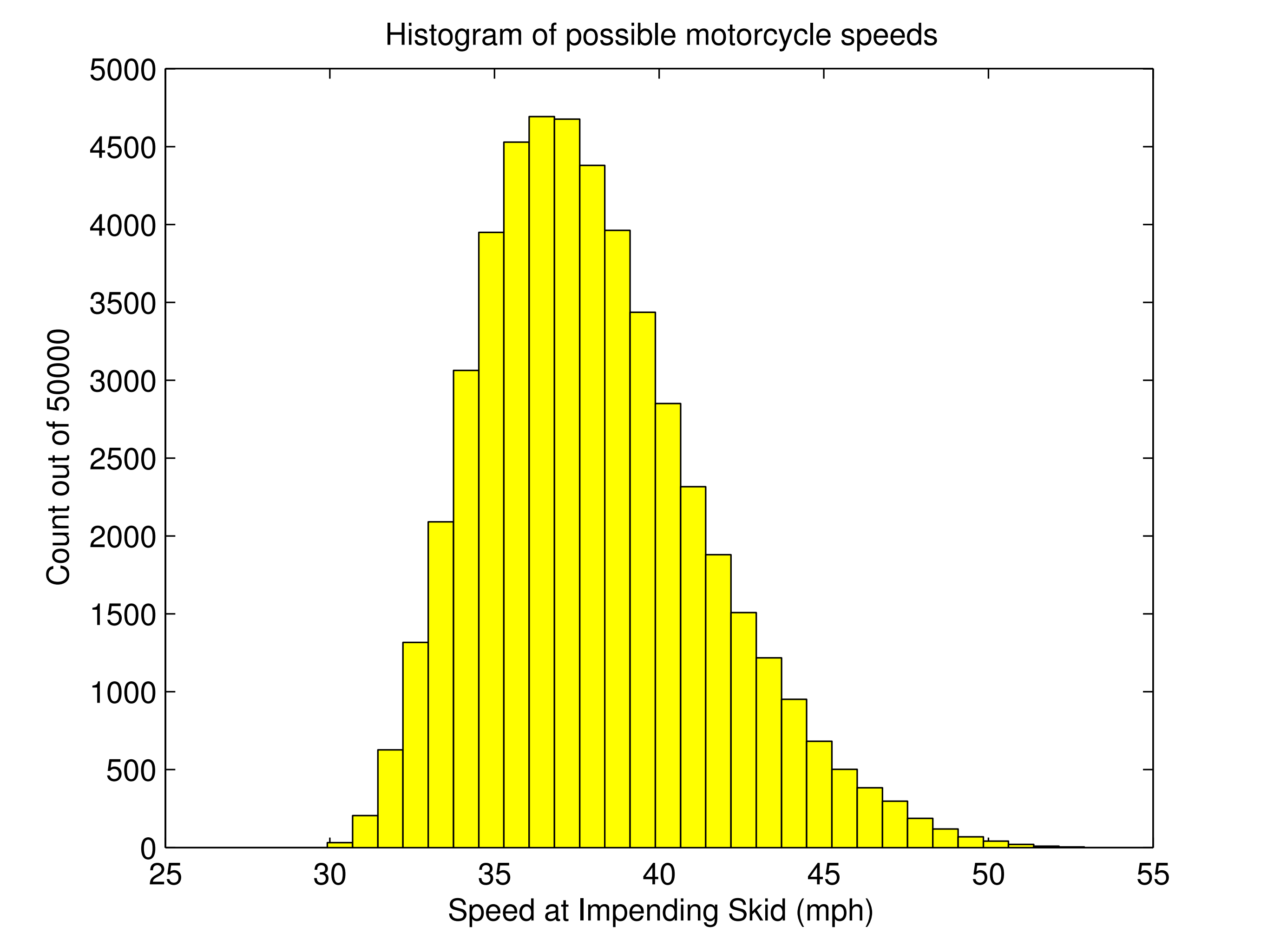
Variable	High	Low
θ	50	30
I	553,830	453,830
r_{turn}	26.1	22.1
$f_{motorcycle}$	0.8	0.6
$f_{lateral}$	0.7	0.5

Variable	High	Low
f_{post}	0.6	0.5
w_{cycle}	697	637
s	15	13
Δt	0.200	0.100
r_{hit}	252	246

50,000 samples were taken from uniform distributions.

All ranges based on "expert" opinion.

Monte Carlo Results



Initial speed was between 33.1 mph and 44.2 mph.

Discussion and Conclusions

What Does This Show?

- Including variation in the inputs still lead to a viable analysis
- Accounting for external impulse enables solution
- Thorough scene documentation is needed to perform reconstruction
- Using the actual vehicles makes for a compelling case
- Rotational motion is not coupled to translation
- The M/C has angular momentum even when traveling straight (depends on radius).

Limitations

- Planar impact mechanics solution: All vertical projections are considered negligible.
- May be a prohibitive technique without using the actual vehicles and being able to shut down the road.
- Expensive in time and manpower.

Conclusions

1. The M/C was traveling nearly 31 mph at impact.
2. The M/C was likely traveling 42 mph at the beginning of the skid.
3. The sight distance provided ample time for the driver of the truck to see the M/C.
4. Evidence suggests the headlight of the motorcycle was on at the time of the crash.
5. The scrape marks, post impact travel, and the fact that the bike was wedged under the right front tire indicates the truck was moving forward at the time of impact.
6. Analysis resulted in a conviction of the Mack truck driver.

References

[1] W. Bartlett, "Conducting monte Carlo analysis with spreadsheet programs," *SAE Technical Paper Series*, no. 2003-01-0487, 2003.

[2] F. P. Beer and E. R. Johnston Jr., *Vector Mechanics for Engineers: Dynamics*, 5th ed. McGraw Hill, 1988.

[3] J. G. Daily, N. Shigemura, and J. S. Daily, *Fundamentals of Traffic Crash Reconstruction*, ser. Traffic Crash Reconstruction. Jacksonville, Florida: Institute of Police Technololgy and Managment, University of North Florida, 2006, vol. 2.

Any Questions?