

Critical Speed Yaw Analysis and Testing

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Critical Speed Yaw

- What is a “yaw”?
- What does the word “Critical” mean?
- How do we get “Speed” out of this?

Definitions

- Yaw refers to the orientation of the vehicle.
- As a vehicle goes around a turn, it is in a “yaw” because it is rotating around the z-axis.
- As the vehicle reaches maximum performance, it will develop a slip angle and the rear tires will begin tracking outside the corresponding front tires.
- When this happens, the heading of the vehicle is not co-linear with the velocity vector of the vehicle.
- The evidence showing this are the tire scuffs on the road.

Definitions, cont'd.

- “Critical” refers to the idea that we may identify the lateral acceleration as the maximum available friction on the road.
- We obtain this friction information with a test skid.
- We may have to correct our test skid information for grade or superelevation.
- The **ONLY** way a vehicle gets onto a critical speed yaw is through the application of an inappropriate steering input (over correction)!

Speed Calculation

- To determine a speed from a critical speed yaw situation, we need the radius of the curved path of the vehicle center of mass and the lateral acceleration of the vehicle.
- Lateral acceleration comes from skid testing.

$$S = 3.86\sqrt{rf}$$

Road Evidence

In order to properly analyze a critical speed yaw situation, we must recognize the tire mark evidence left behind.

Note the diagonal striations and uneven edge of the tire marks left in the following photos.

The rear tires begin tracking outside the front tires.

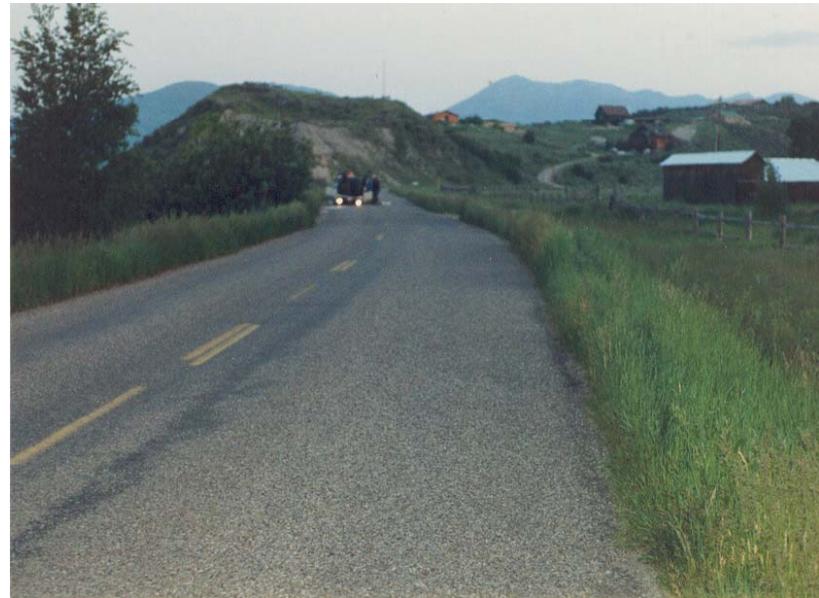
Yaw Marks (Scuffs)

- Tire is sliding and rotating
- A yaw results when the velocity vector and heading of the vehicle are not co-linear.
- If the yaw is the result of inappropriate steering input, then the yaw may be a “Critical Speed Yaw”, and may be used for speed analysis in a simple, straightforward manner.
- This photo illustrates a critical speed yaw. Notice the rear tires track outside the corresponding front tires.



Yaw Marks, Cont'd.

- This crash began with critical speed yaw marks on the shoulder
- We can use shoulder drag to get a conservative speed estimate.
- Bellion (SAE 970955) suggests using 66% of the shoulder drag and 33% of the road drag to determine lateral f .



Yaw Marks, Cont'd.

- The Yaw begins to transition to a side skid. We may no longer treat it as a critical speed yaw.
- A spin analysis would be more appropriate



Yaw Marks, Cont'd.

- Another example of transitioning yaw marks-
- If these marks were analyzed as critical speed yaws, the speeds would be way off – probably on the high side...A spin analysis is appropriate here.



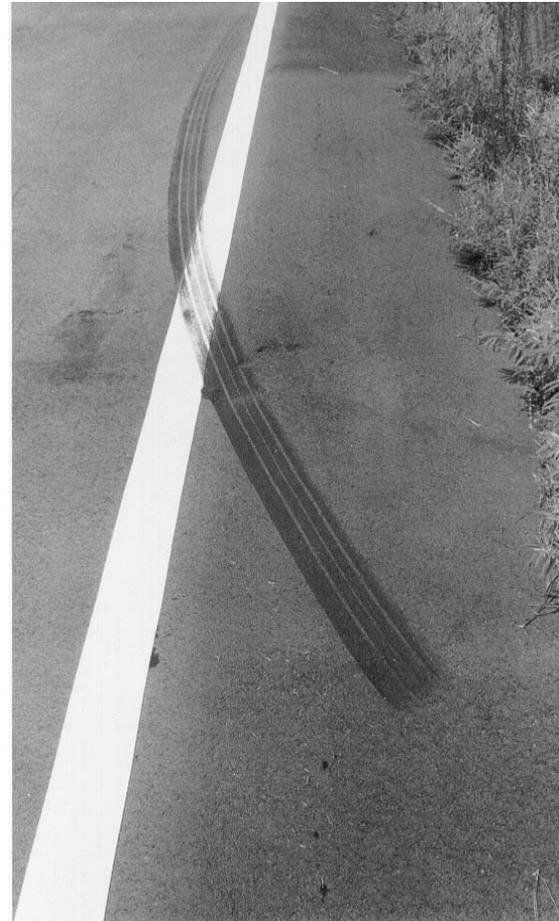
Yaw Marks, Cont'd.



- Note the transition from CSY to spin...

Yaw Marks, Cont'd.

- This is a curving tire mark, but does not indicate a yaw.
- This is a curving acceleration scuff.
- Other curving marks may also look like critical speed yaws, but are not.
- Is there a way to examine this question?
- We will examine the evidence...



Yaw Marks, cont'd.

- This is a CSY mark left with no braking.
- Note the direction of the striations in the tire mark.



Yaw Marks, cont'd.

- This is a CSY mark left with braking.
- Note the direction of the striations in the tire mark.
- As the brake force increases, the angle of the striations approaches parallel.



Yaw Marks, cont'd.

- This is a CSY mark left with ABS braking.
- Note the direction of the striations in the tire mark change as the ABS system cycles.
- The average lateral acceleration is a little below the ABS drag factor.



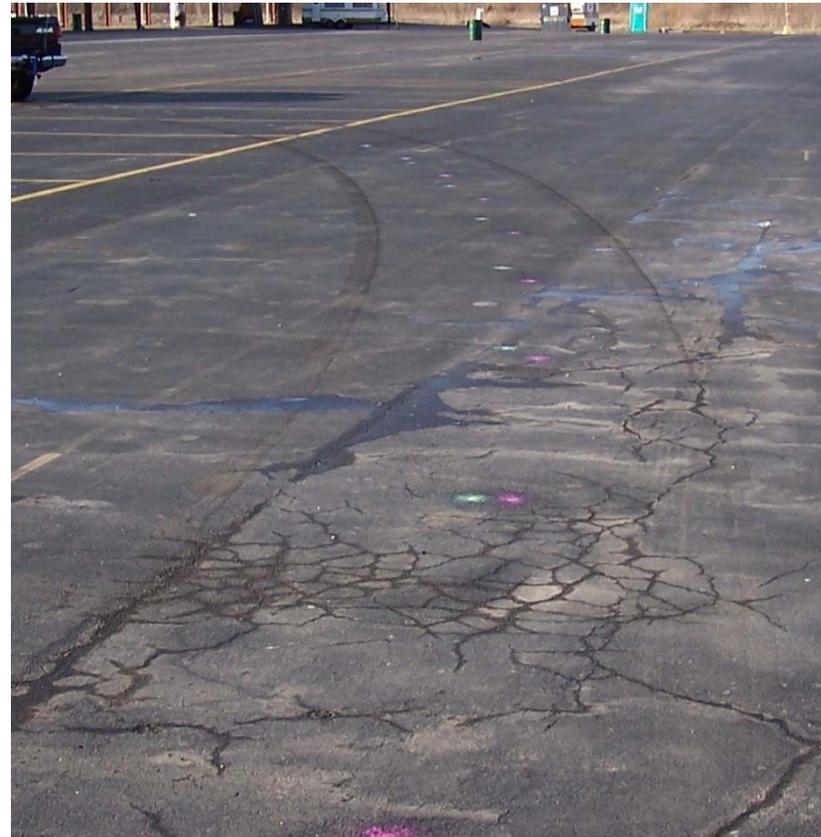
Yaw Marks, cont'd.

- Transition from CSY to full, non-ABS braking.
- This evidence may be left by the panicked, untrained driver who inappropriately steers and then brakes to maximum.
- The yaw portion follows the CSY model.



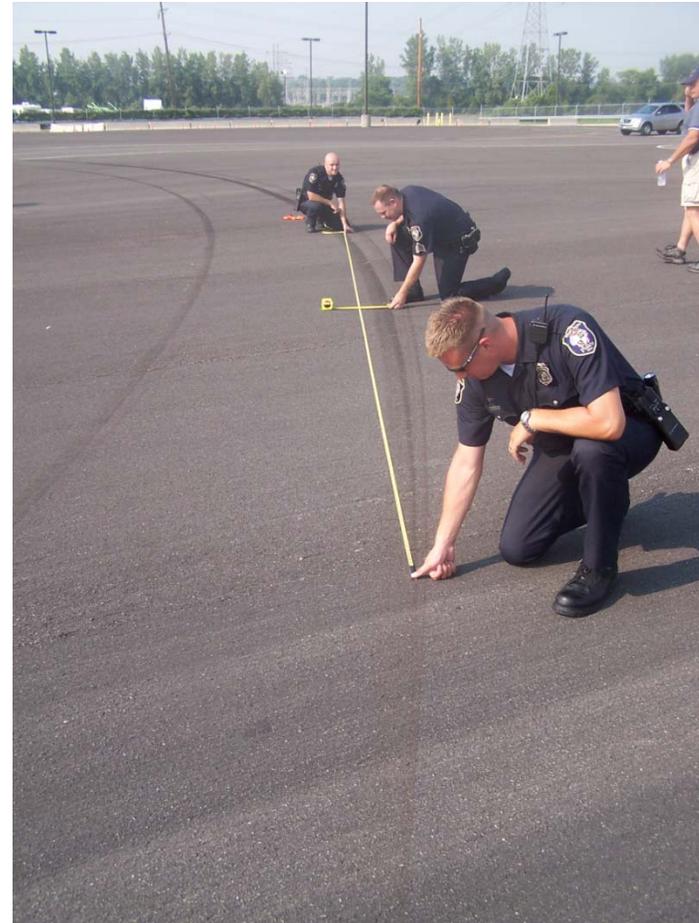
Yaw Marks, cont'd.

- Controlled braking and turning by a trained driver. Rear ABS only.
- The inside mark is a skid, with longitudinal striations.
- The rear tires track outside the corresponding front tires.
- The outside tire marks show diagonal striations.
- May NOT use the CSY analysis with a full drag factor.
- Lateral acceleration is significantly lower than the full drag factor.
- Evidence does NOT support the CSY analysis. A skid analysis would be more appropriate.



Radius Determination

- In order to obtain a path radius, we first measure a chord of the arc left by the outside front tire.
- A chord is any line that cuts the arc at two points.
- The chord beginning is right after the rear tire mark crosses over the front tire mark.
- Chord lengths should be between 30 and 50 feet. The tape must be pulled tight as it must be *straight!*



Radius Determination, cont'd.

- We measure the chord from the OUTSIDE edge of the outside darkest tire mark.
- This is usually the outside front tire.
- The chalked “V” mark is where the measurement is being taken.



Radius Determination, cont'd.

- After we have laid down the chord measurement, we have to determine a middle ordinate.
- The middle ordinate divides the chord in half and extends out to far the edge of the tire mark.
- For a 30 foot chord, measure the middle ordinate to the nearest 1/8 inch.
- Choose a chord length that will result in a middle ordinate of at least 6 inches.
- Measure from the outside edge of the tape to the outside edge of the tire mark.



Radius Determination, cont'd.

- Once we have measured the first chord and middle ordinate, we measure a second.
- Begin at the end of the first chord and measure a similar chord and middle ordinate as indicated in the photo.
- This will enable us to calculate two radii, which will help us to determine the deceleration in the yaw.



Radius Determination, cont'd.

- Once we have chord and middle ordinate measurements, we may calculate the radius of the outside front tire mark with the following equation:
- To get the CM radius, subtract $\frac{1}{2}$ of the track width from this calculation.

$$R = \frac{c^2}{8m} + \frac{m}{2}$$

Measuring Grade

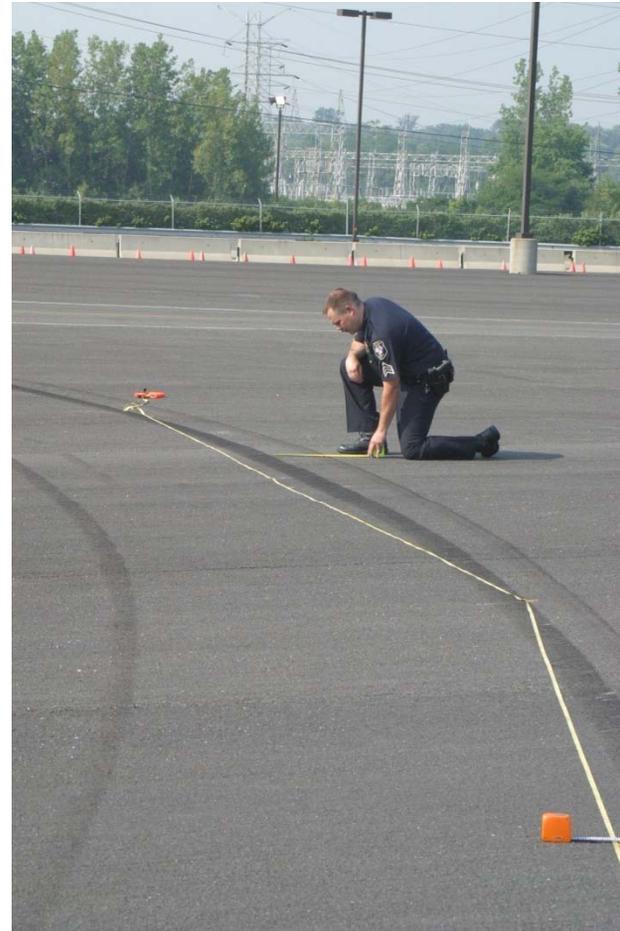


Measuring Super-elevation



Measuring Out-tracking

- Find the point within the first two chords where the rear tire tracks furthest outside the front tire.
- Measure this out-tracking from the outside edge of the front tire mark to the outside edge of the rear tire mark.
- Take this measurement and divide it by the wheelbase of the vehicle. Keep all units consistent!
- If this calculated ratio, ρ (rho) is greater than 0.35, analyze the marks as a spin.



Skid Testing

- In order to get the proper drag factor for a critical speed calculation, we must do test skids, preferably with an exemplar vehicle.
- Results between tests must be consistent.
- If there is apparently no braking in the CSY, then we may use an ABS generated drag factor if the test vehicle is so equipped.
- If there appears to be ABS braking along the CSY, then use a locked and sliding drag factor.

Critical Speed Yaw?

- We need a tool we can use to determine if the vehicle is actually in a critical speed yaw.
- If we look at the dynamics of a vehicle in this maximum steering situation, we would expect the vehicle to decelerate longitudinally as it progresses through the yaw.
- If we can quantify this deceleration at a nominal value, then we may look at any particular case and determine if its deceleration fits into the model.
- Much testing over the years suggest a nominal value for yaw deceleration on a dry, hard-surfaced road, is about 0.20g.

Critical Speed Yaw?

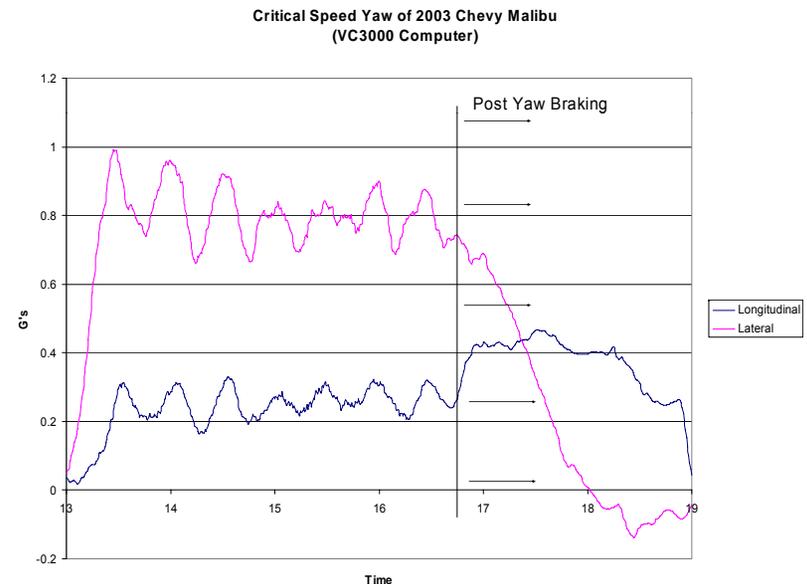
- We may examine the deceleration in the CSY and see if it is within a reasonable range.
- At higher speeds, tires will saturate at smaller steering angles and the vehicle will undergo lesser deceleration.
- Testing at IPTM has shown the deceleration may vary from as little as 0.0 up to 0.35 for vehicles in CSY situations.
- In addition, the CSY is considered a *steady state* situation. We will examine this premise next.

Steady State

- Just what does “*steady state*” mean?
- If we look at the CSY speed equation, we see it is a function of both radius and lateral acceleration.
- *Steady State* means these variables are essentially constant over the time and distance of interest.
- In the case of a CSY, we make the assumption over the first chord distance, and hence time, that these two variables are a constant value.
- Because of vehicle inertial properties, it takes both time and distance for the vehicle to “setup” in the CSY.
- This is why we recommend beginning chord measurements when the rear tire is clearly tracking outside the corresponding front tire.

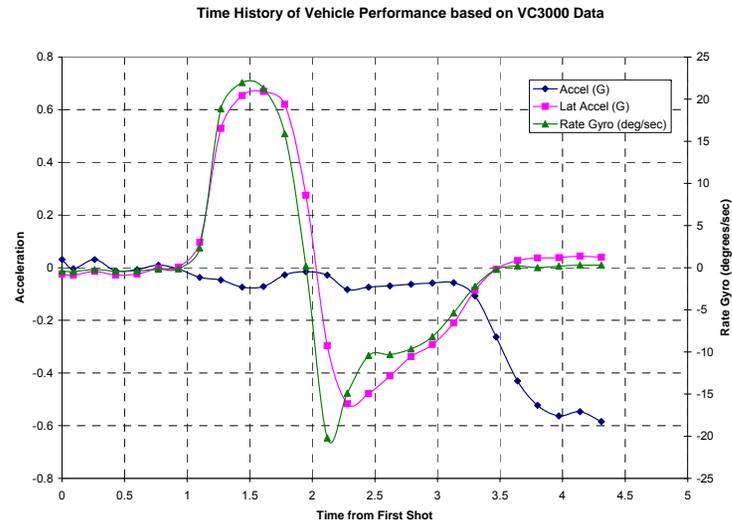
Steady State

- Consider the graph on the right which shows both lateral and longitudinal acceleration in a CSY situation.
- We may see the *average* accelerations are quite constant over several seconds.
- In this case, which was a CSY from 30 mph, the accelerations held constant for about 3 ½ seconds.
- This translates into a distance of a little less than 154 feet.
- Two 30 foot chords fit this distance with room to spare.
- This is an example of *steady state* behavior.



Steady State

- Consider the graph on the right which shows both lateral and longitudinal acceleration in a *Lane Change* situation.
- We may see the *average* accelerations are NOT constant over time.
- A *Lane Change* is NOT a steady state maneuver.
- It MAY NOT be analyzed with the traditional CSY equation!



Critical Speed Yaw Analysis

- If we measure two chords and two middle ordinates, we may calculate two radii.
- If the second chord is measured from the end of the first chord, we may calculate the difference in radius between the first chord and the second chord.
- The second radius calculated should be smaller than the first if the vehicle is slowing.
- If you cannot measure at least two 30 foot chords, then it is best to look for another analysis.

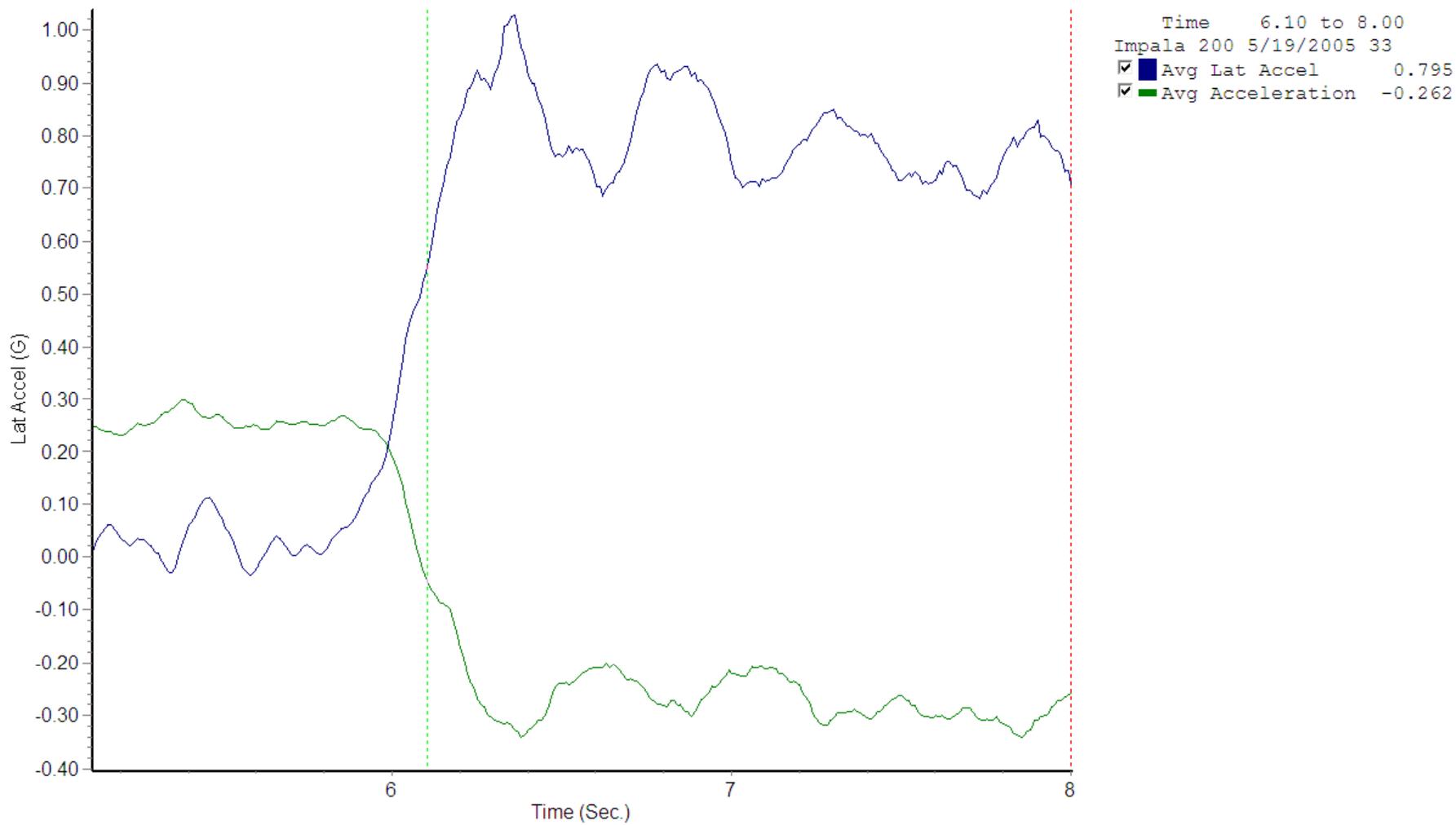
Critical Speed Yaw Analysis, cont'd.

- Calculate a speed for each radius.
- Use the following equation to determine the deceleration in the yaw:
- The distance d in the equation is from one middle ordinate to the other.

$$f = \frac{S_o^2 - S_f^2}{30d}$$

Critical Speed Yaw Analysis, Cont'd

- The graph on the next page shows typical lateral and longitudinal accelerations from a vehicle in a CSY situation.
- As speeds get higher, the longitudinal acceleration usually decreases.
- At higher speeds, tires saturate with less steering angle



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A “Rule of Thumb” Check

- As CSY speeds increase, the tire mark usually gets narrower.
- Top Photo: 71 mph calculated
- Bottom Photo: 43 mph measured and calculated.
- Higher speeds and wide CSY marks are usually not compatible, regardless of tread width.



Critical Speed Equations

- Radius Equation

$$R = \frac{c^2}{8m} + \frac{m}{2}$$

- Critical Speed Yaw Equation

$$S = 3.86\sqrt{rf}$$

- Longitudinal Drag Factor Equation

$$f = \frac{S_o^2 - S_f^2}{30d}$$

Critical Speed Yaw Test Results

- The following table summarizes CSY testing done in several Crash Reconstruction Classes.
- Each CSY was analyzed in the manner presented in the program, using the equations from the preceding slide.
- As we see, the results are consistent and accurate in showing vehicle speed in the CSY.

CSY Test Results

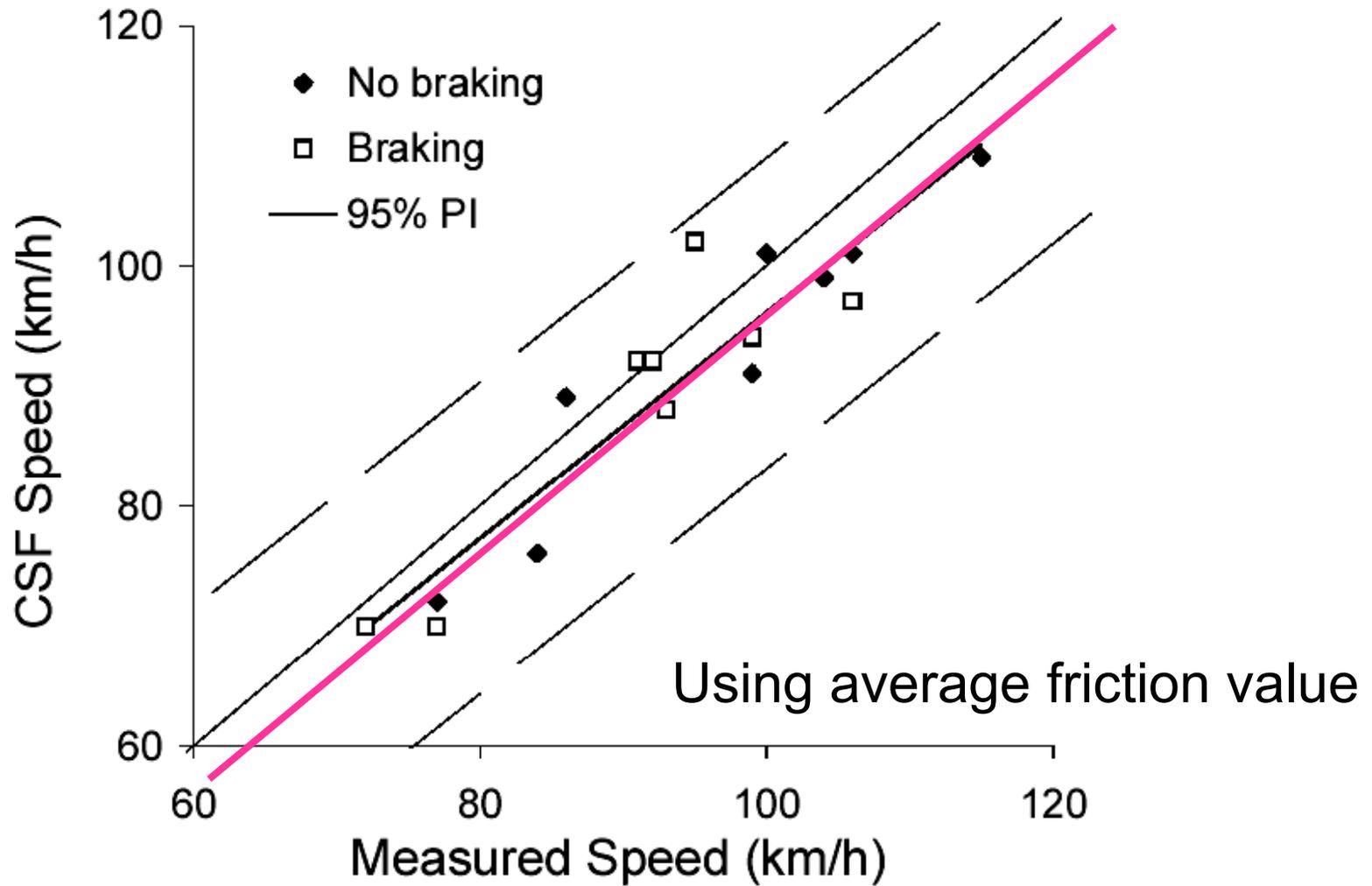
Date	Location	Corrected Radius R₁ / R₂	Drag factor from Skid Test	<u>Longitudinal</u> f Calculated / Measured	Calculated Speed, MPH	Radar or Integrated Speed
July 2002	Wisconsin	84.29 / 71.29	0.83	0.18/0.18	32.28	33
Nov. 2003	Minnesota	123.2/108.9	0.77	0.18/0.181	37.59	38
Nov. 2003	Minnesota	105.4/92.6	0.77	0.16/0.19	34.77	36
July 2003	Ohio	73.74/53.01	0.79	0.27/0.26	28.91	29.5
Oct. 2003	Wyoming	213.6/193.6	0.64	0.21/0.21	45.13	46
Oct. 2003	Wyoming	112.4/94.5	0.64	0.18/0.20	32.73	33
April 2004	New Mexico	110.5/100.3	0.77	0.13/0.13	35.61	36
Oct. 2004	Wisconsin	125.3/116.3	0.89	0.13/NA	40.75	41
May 2005	Illinois	82.46/71.16	0.78	0.15/0.26	30.96	33

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Other Testing: Cliff et al. 2004-01-1187

- “Yaw testing an instrumented vehicle with & without braking”
- 91 Honda Accord
- Yaws with and without light braking
- Measured to outside edge of tire marks
- Used 10m chords starting at first visible mark
- Subtracted $\frac{1}{2}$ track width to get cg path radius (neglected slip-angle correction)

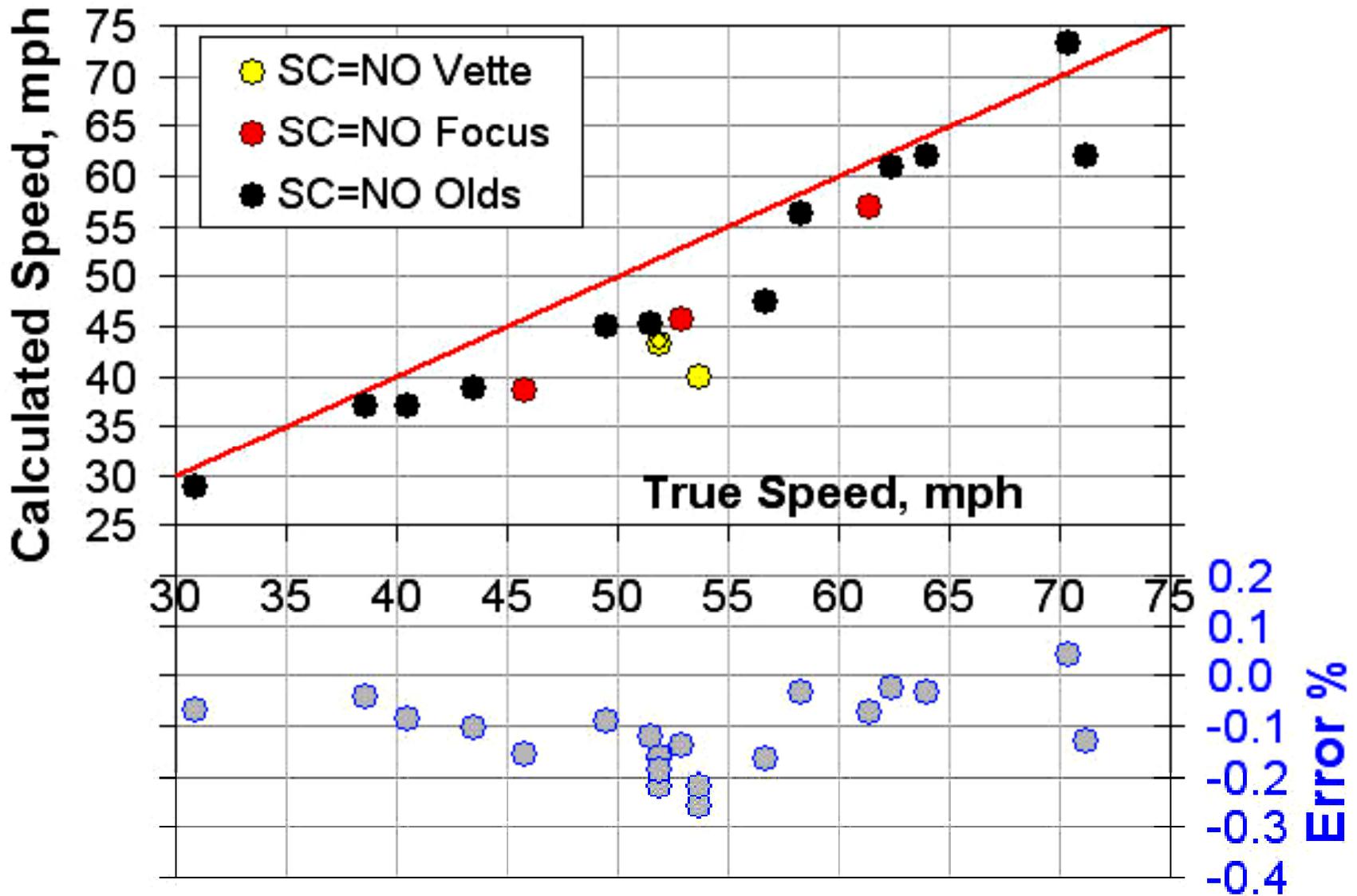
Other Testing: MacInnis 2004-01-1187



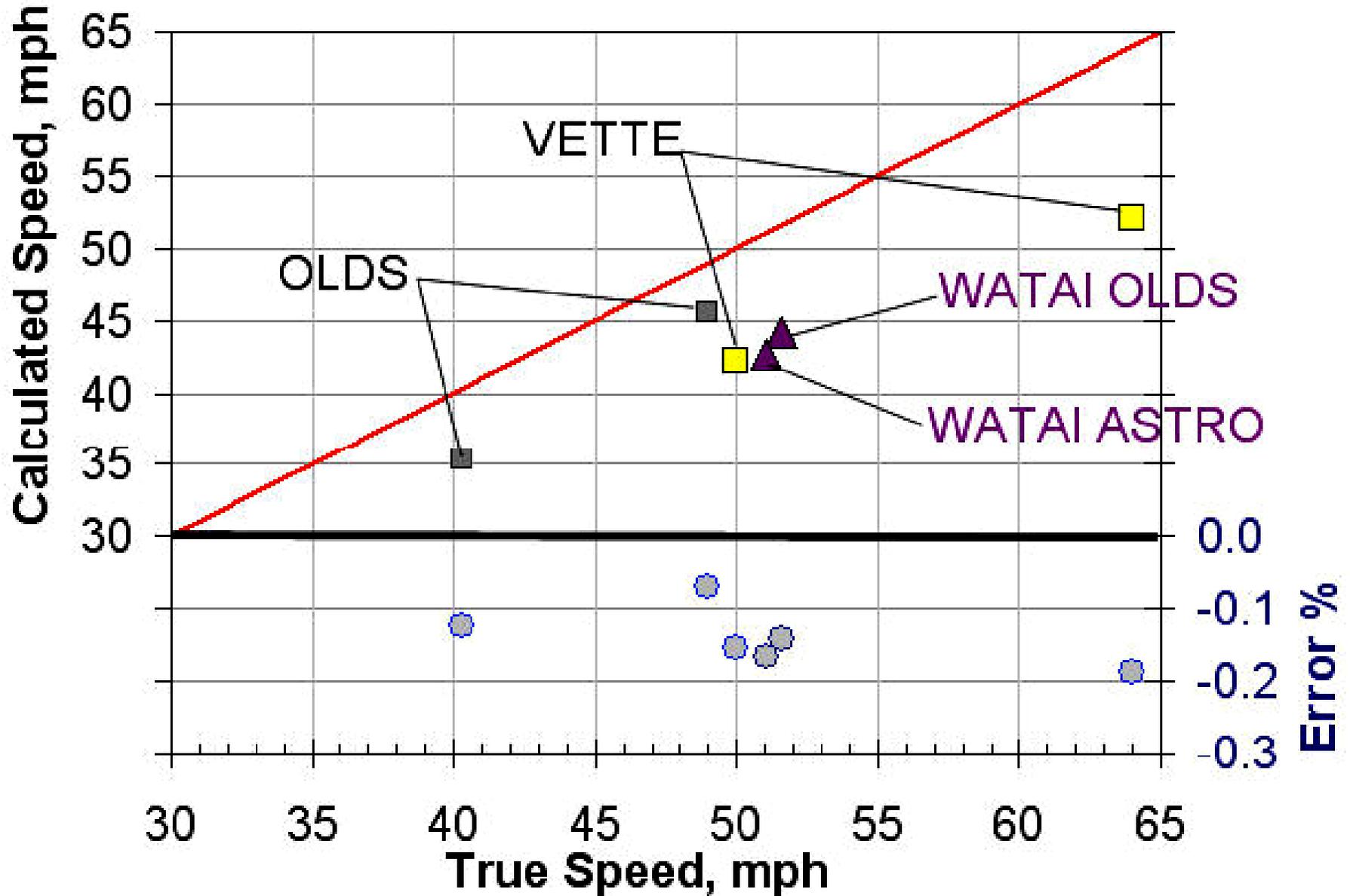
Testing by Wade Bartlett and Bill Wright

- Conducted over a couple years
- Various locations & vehicles
- Speed based on GPS, Vericom, 5th wheels
- 30-ft chords, average deceleration during skid

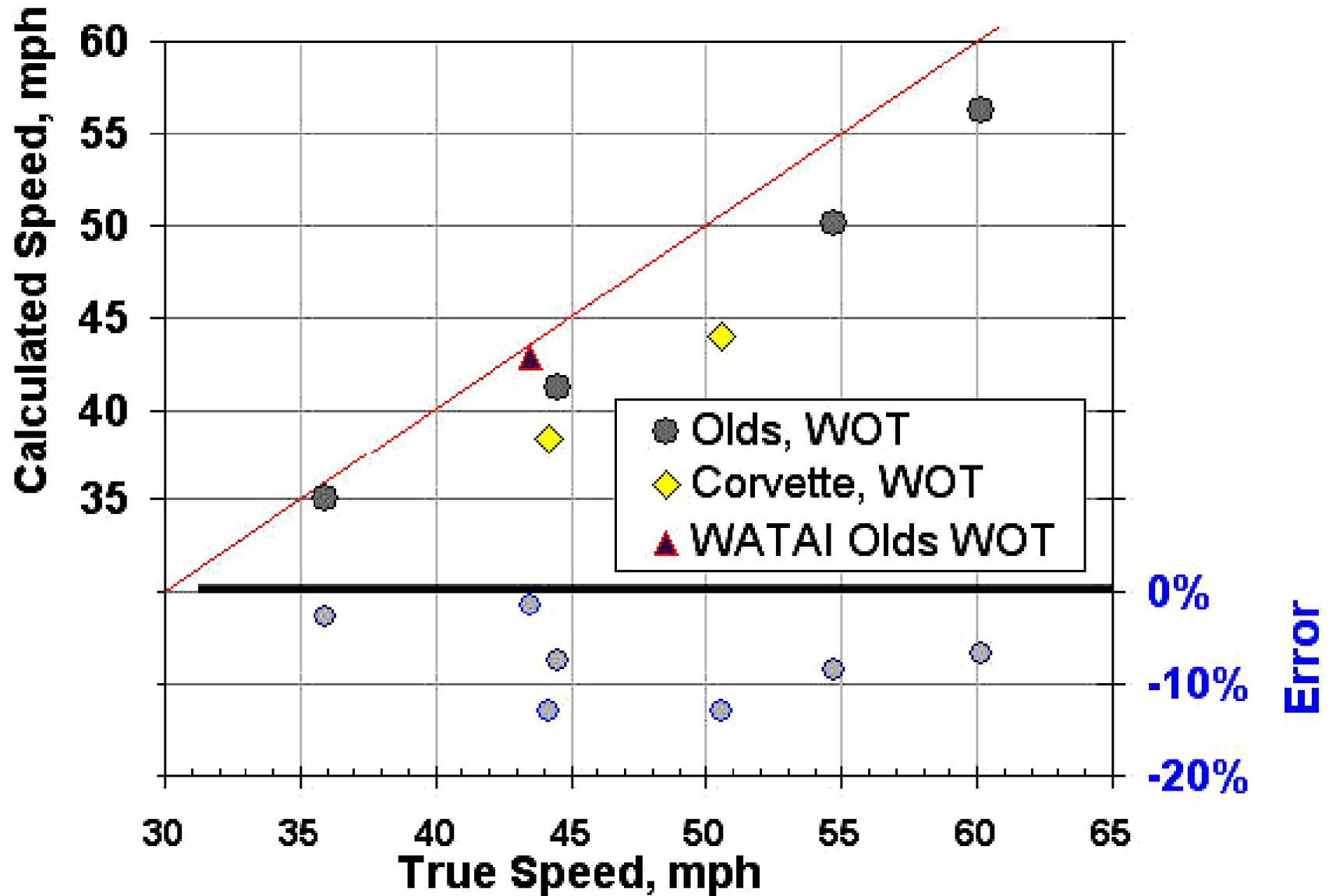
First 30-foot Chord – Avg.f



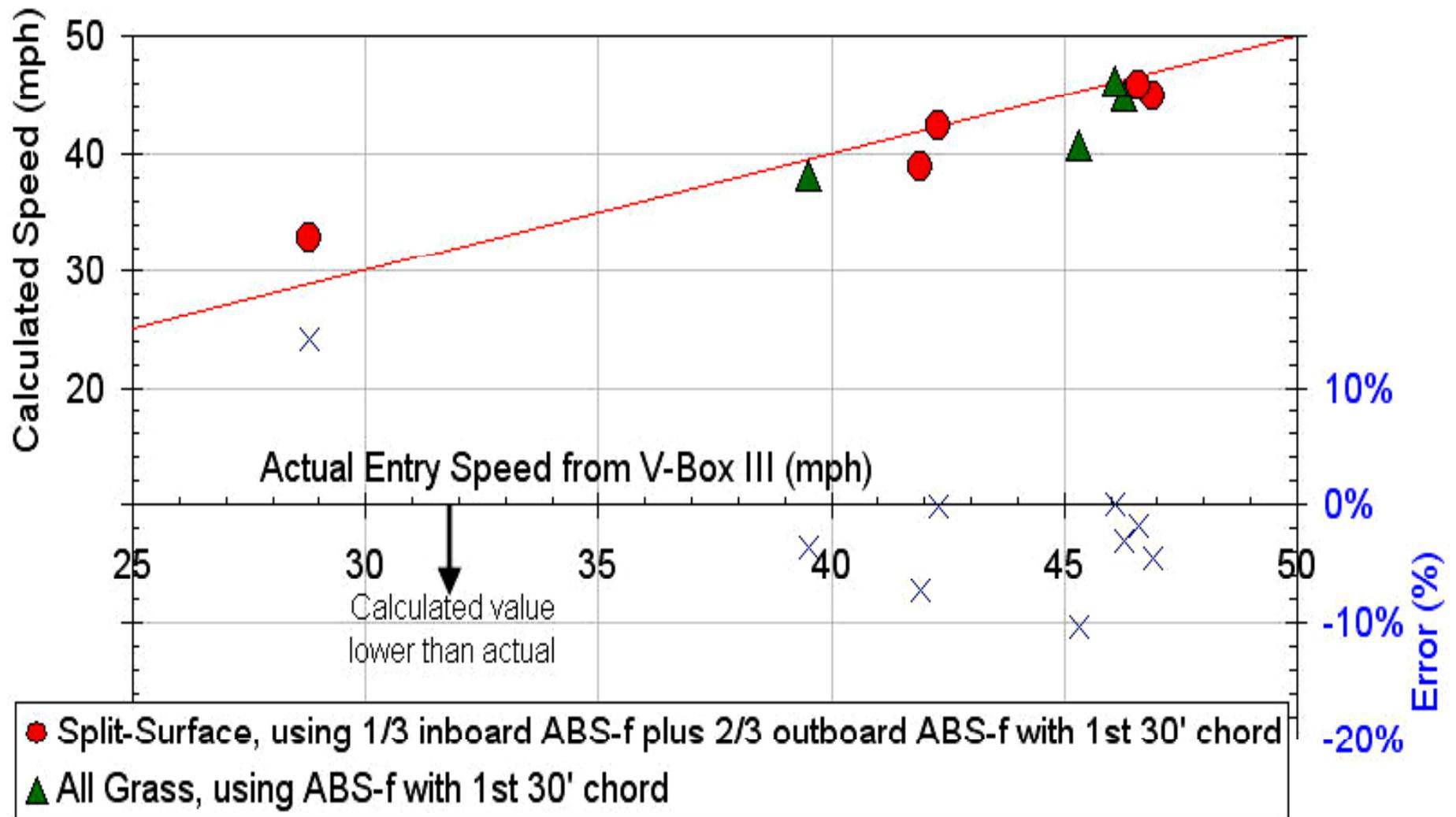
CSY with ABS Braking



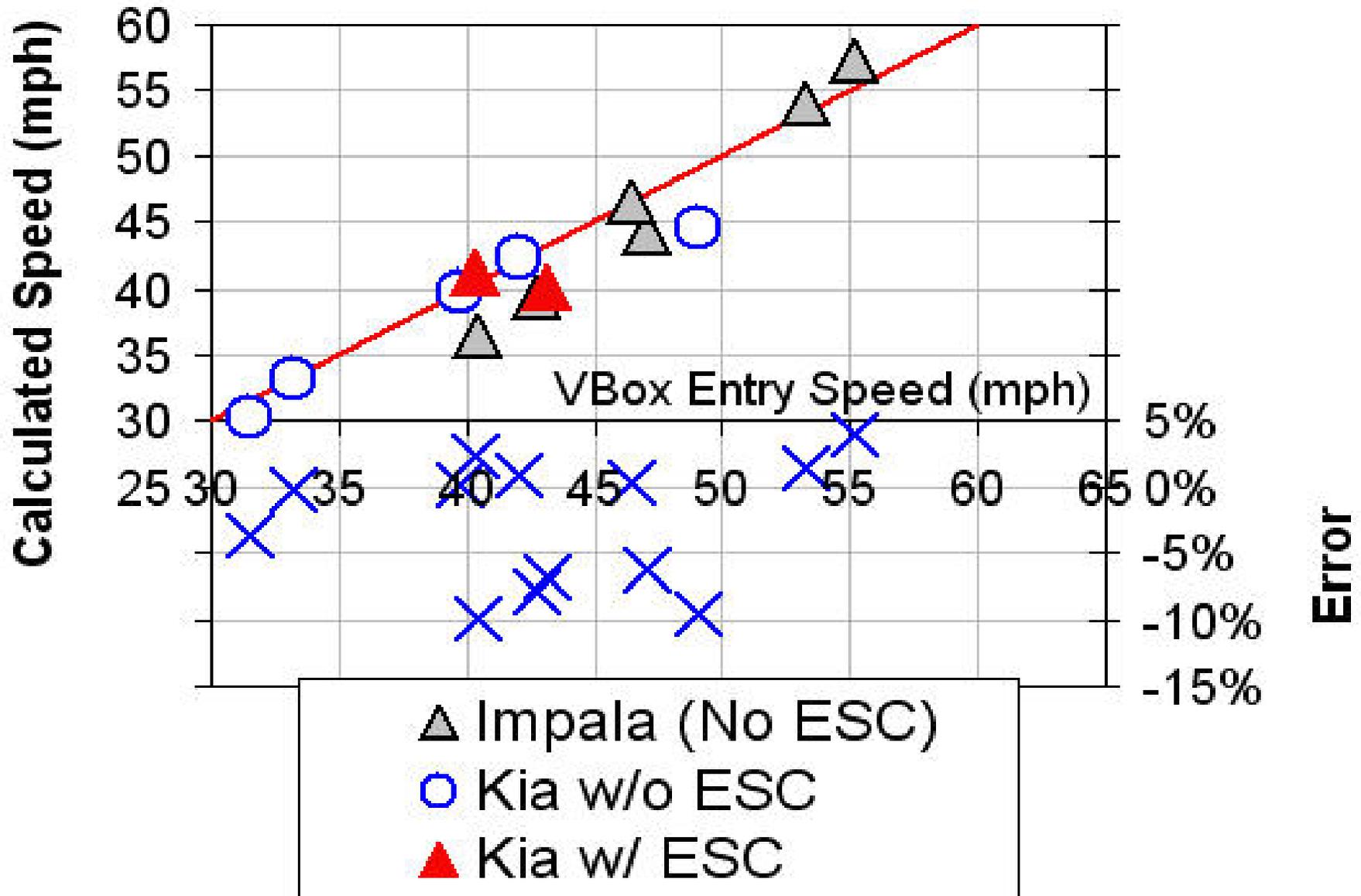
WOT on Pavement



Split Coefficient Surface



Gravel – with & w/o ESC Using ABS-friction value



Sensitivity Analysis

- The Critical Speed analysis depends upon two variables:
 - The Radius of the path of the Center of Mass
 - The lateral acceleration factor for the vehicle

How sensitive is the overall speed computation to these two variables?

Effect of Radius

- The radius of the tire mark is calculated from the chord and middle ordinate measurements.
- Of these two, the radius equation is most sensitive to the investigator's ability to measure the middle ordinate.
- We recommend the middle ordinate be measured to the nearest $\frac{1}{8}$ inch for a 30 foot chord.

Effect of Radius, cont'd.

- Consider the following information:
 - The chord is 30 feet
 - The actual middle ordinate measurement is 6 inches or 0.5 feet.
 - The middle ordinate is measured at 5 ½ inches, or 0.458 feet.
 - The actual radius is 225.25 feet
 - The radius from the measurement is 245.86 ft
 - The error is 20.61 feet, or +9.1%

Effect of Radius, cont'd.

- The speed calculated from the actual radius is **50.17 mph** with a nominal drag factor of 0.75.
- The speed calculated from the measurement is **52.41 mph**.
- The difference is 2.24 mph, or 4.4%

Effect of Drag Factor

- We used a nominal 0.75 for the previous calculations.
- If we use a constant radius but vary the drag factor, what is the effect?
- For a radius of 225.25 feet and a drag factor of 0.70, then **S = 48.46 mph.**
- For a radius of 225.25 feet and a drag factor of 0.80, then **S = 51.81 mph.**
- The difference is 3.35 mph, or 6.6%.

Effect of Both Variables

- The radius varies between 225.25 feet and 245.86 feet.
- The drag factor varies between 0.70 and 0.80
- The speed calculated from the low values is **48.46 mph.**
- The speed calculated from the high values is **54.13 mph.**
- The difference is 5.67 mph or 11.3% compared to the nominal 50.17 mph.

Summary

- We have examined the Critical Speed Yaw analysis.
- The technique has proved its usefulness over the years.
- We measure the chord and middle ordinate for the outside front tire immediately after the rear tire has crossed over the front tire mark.
- Use ABS Test skid data if no braking in the CSY. If braking is present, use the locked wheel skid data.
- We must be sure the vehicle is actually in a critical speed yaw.
- Measure the out-tracking and divide by the wheelbase. This result should be 0.35 or less.
- Examine the deceleration of the vehicle.
- A nominal deceleration value for dry, hard roads is 0.20g for the first third of the yaw. Higher speeds result in lower accelerations.
- Be very careful with both the drag factor determination and the middle ordinate measurement!
- With proper care, the method has been and remains viable!