

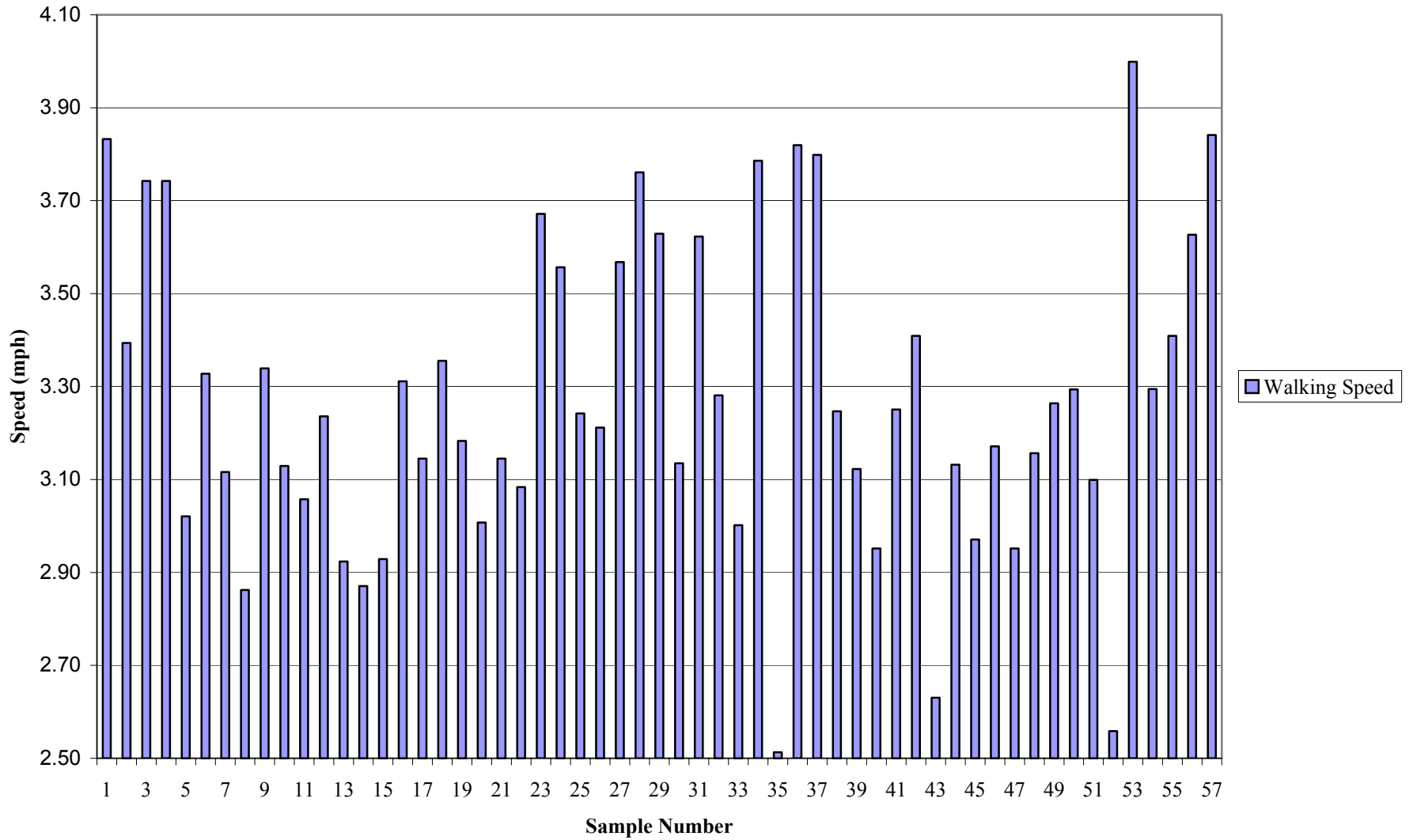
Introduction to Monte Carlo Simulations

Jeremy Daily

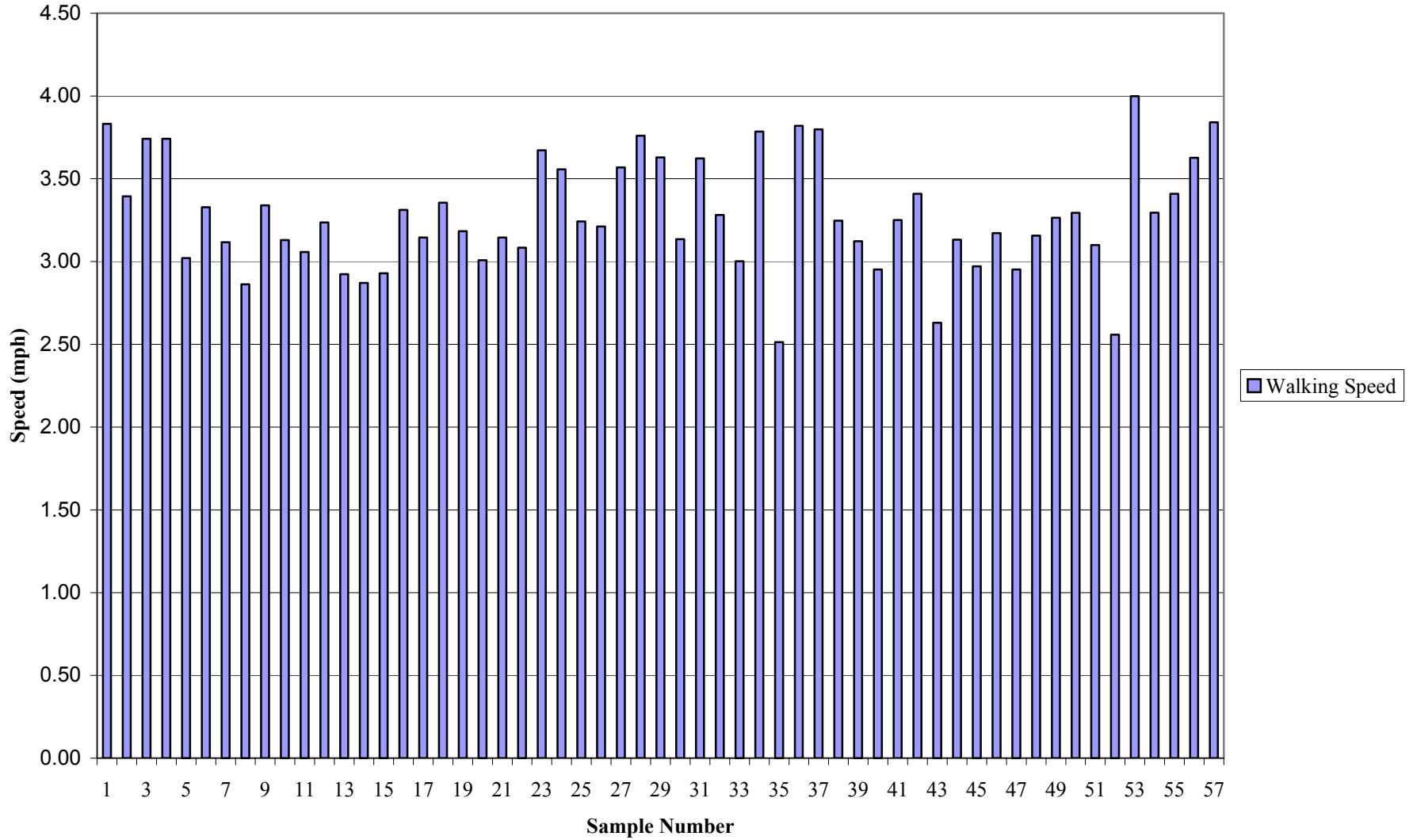
Jackson Hole Scientific Investigations

www.jhscientific.com

Walking Speed



Walking Speed



Statistical Deception

- Graphs are skewed to show some bias
- Look carefully at the labels and axes
- Look at the number of samples (the larger the better)
- If no variation is mentioned, be skeptical
- The biggest culprit—
 - BIASED SAMPLING!!!

Coin Toss

- Flip a coin ten times and count the number of heads
- Construct a table of results

Count											
Number of heads	0	1	2	3	4	5	6	7	8	9	10

Binomial Distribution

- What about 100 tosses?
- What is the probability of getting exactly 50 heads?
- What about the chances of getting 50 heads or lower?

- Theoretical Probability for 10 Tosses:

0 Heads: $P = 0.0010$

1 Heads: $P = 0.0098$

2 Heads: $P = 0.0439$

3 Heads: $P = 0.1172$

4 Heads: $P = 0.2051$

5 Heads: $P = 0.2461$

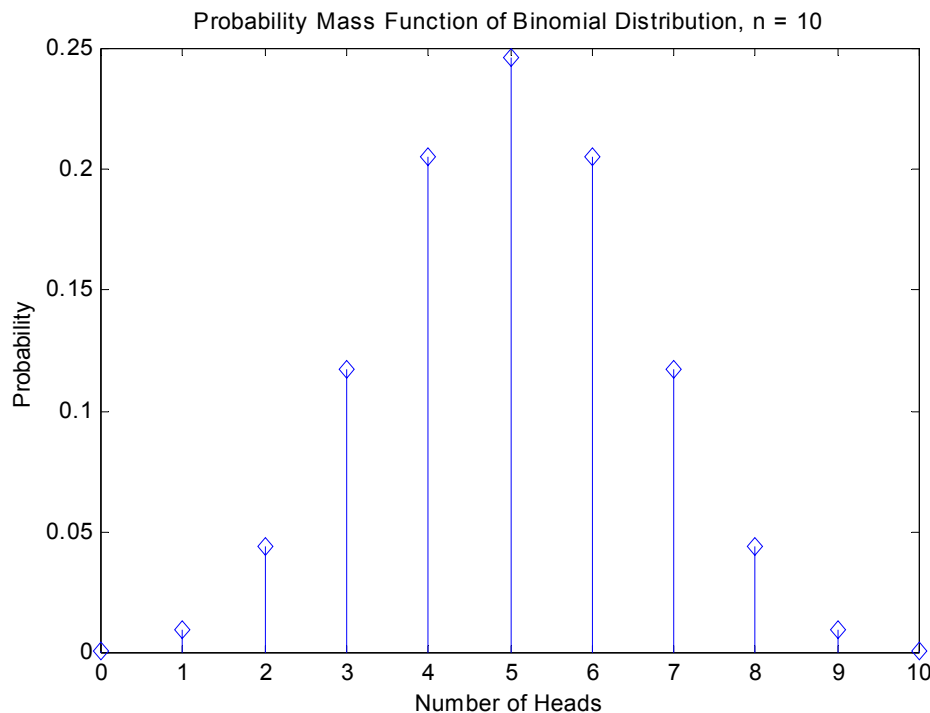
6 Heads: $P = 0.2051$

7 Heads: $P = 0.1172$

8 Heads: $P = 0.0439$

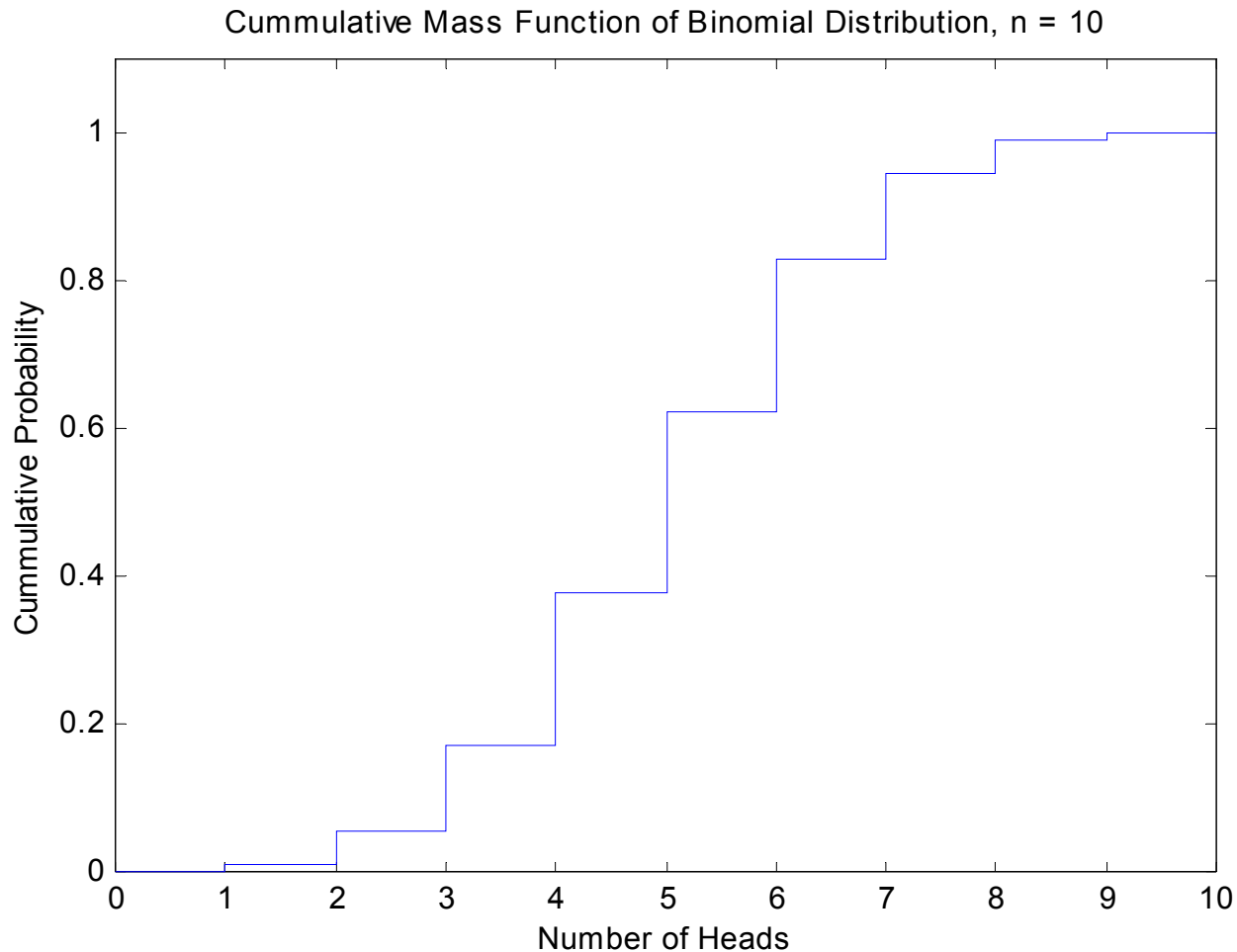
9 Heads: $P = 0.0098$

10 Heads: $P = 0.0010$

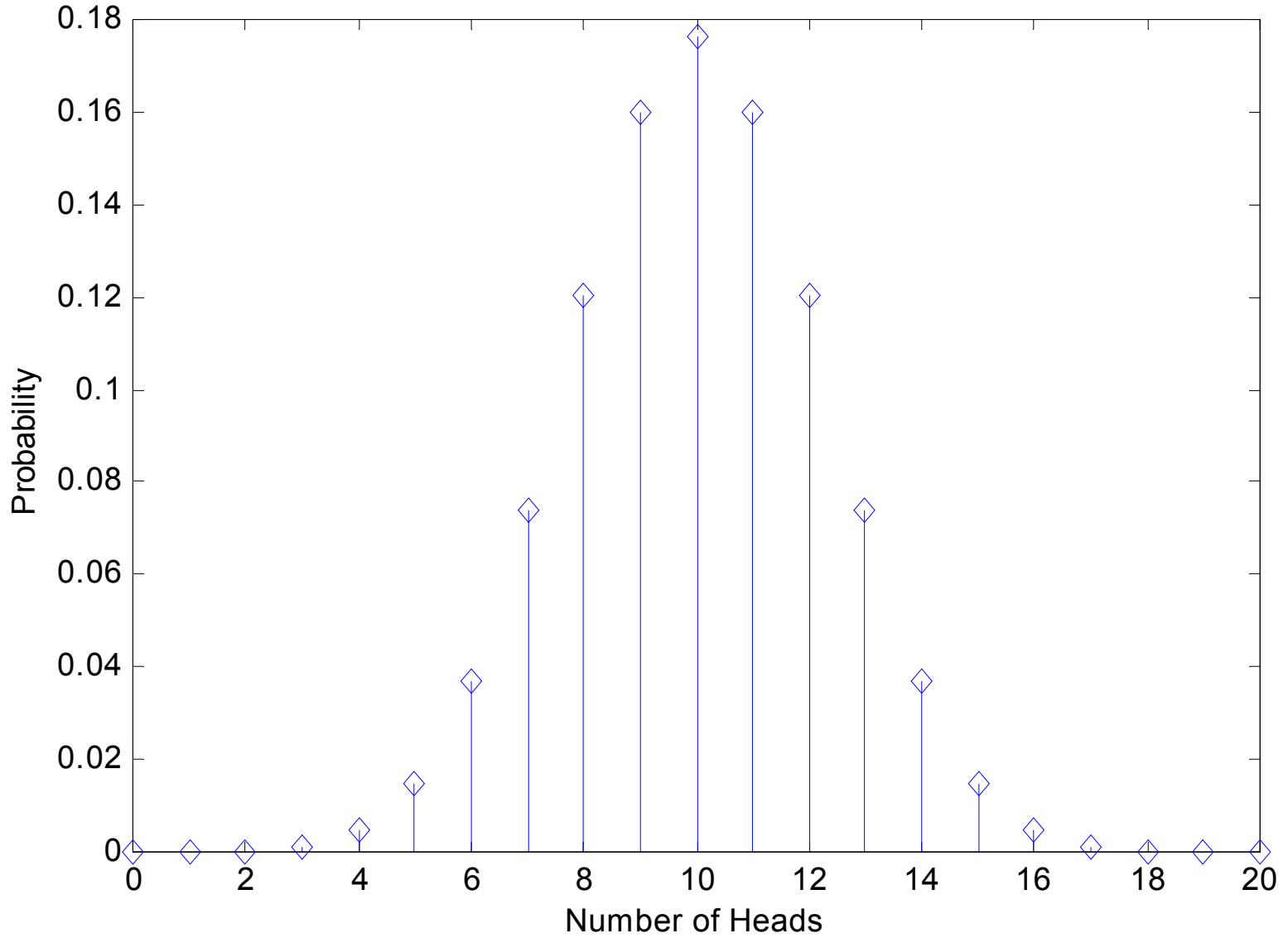


Cumulative Probability

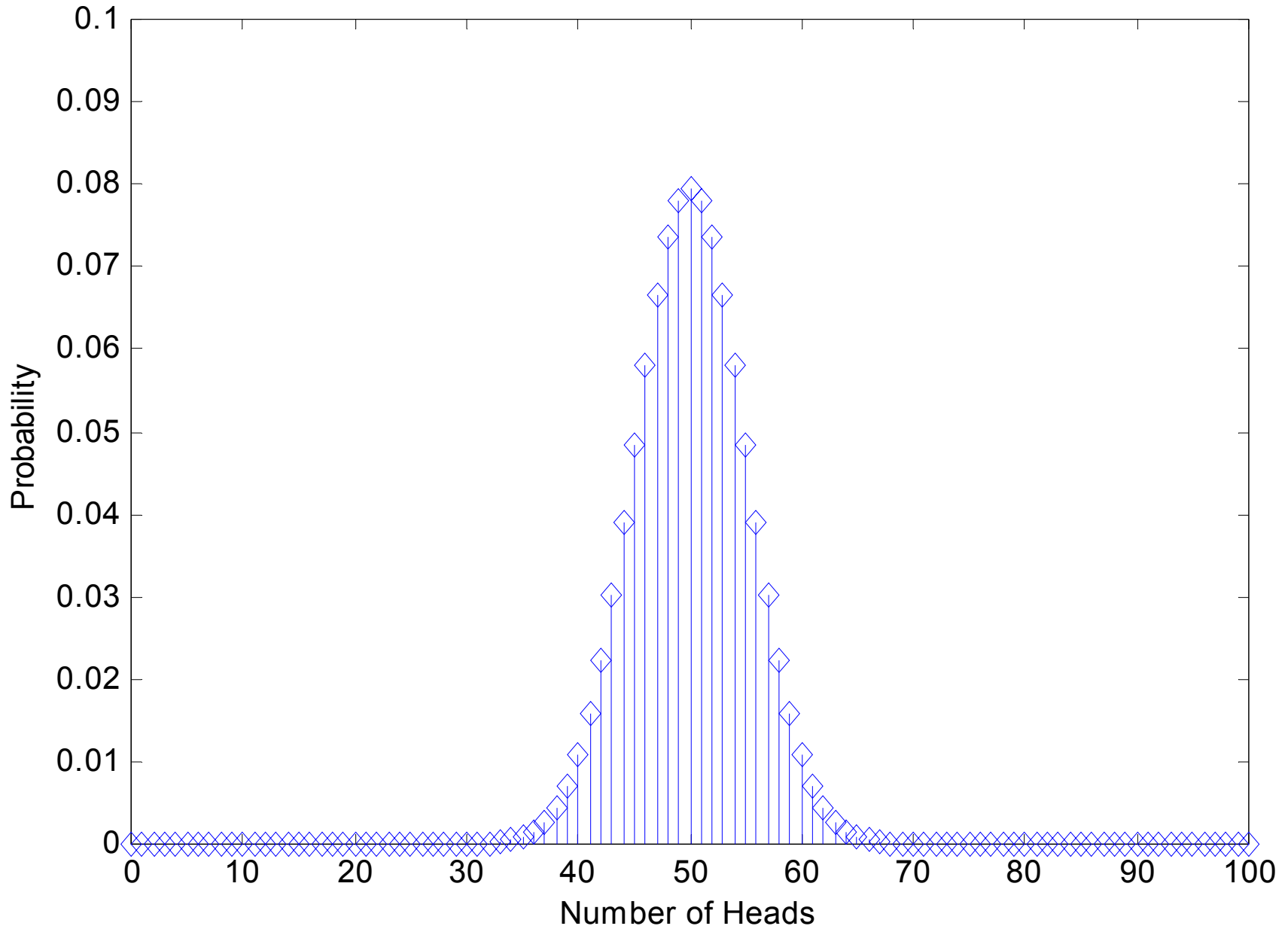
- Gives probability of something being lower than x .



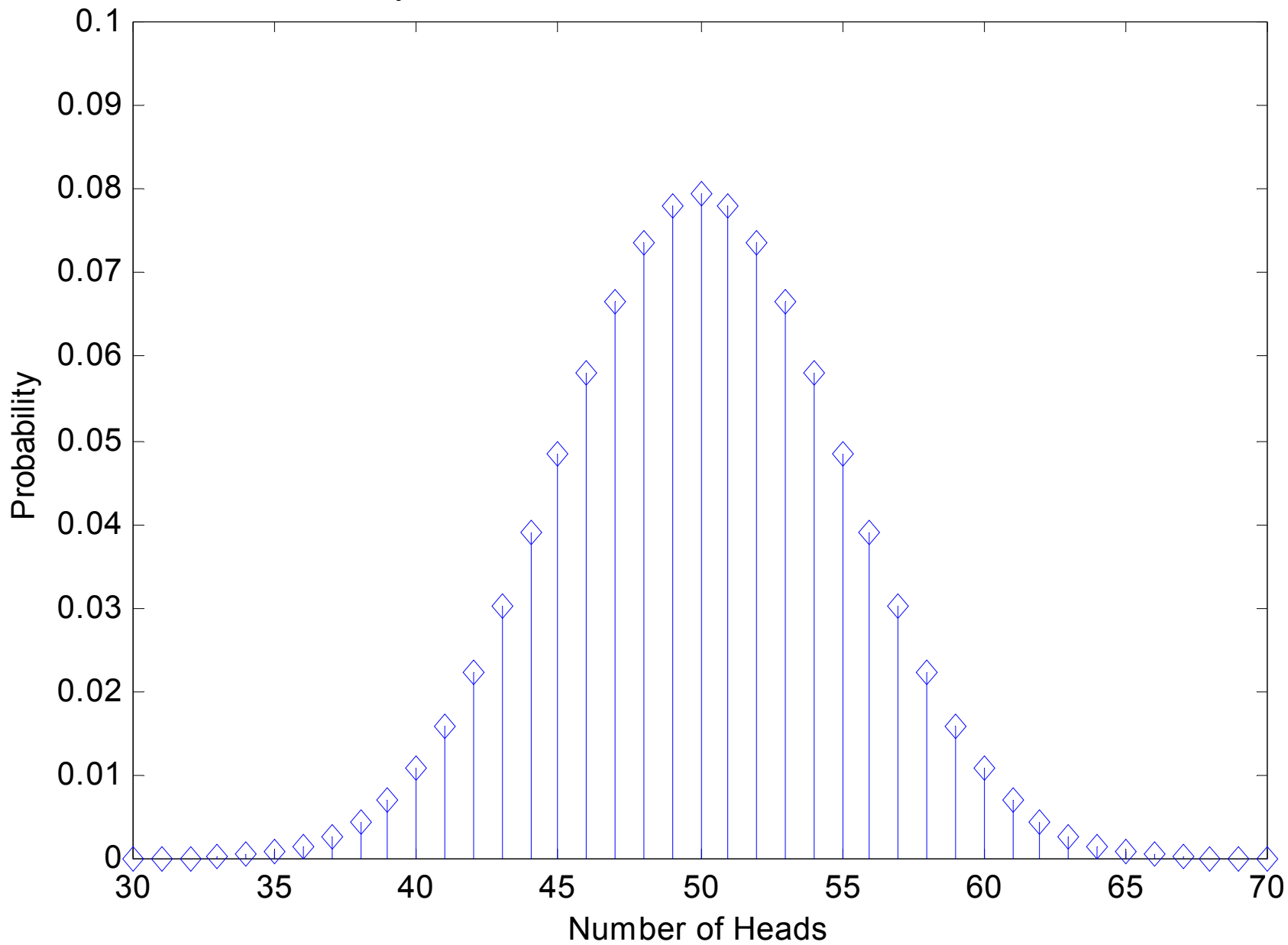
Probability Mass Function of Binomial Distribution, $n = 20$



Probability Mass Function of Binomial Distribution, $n = 100$



Probability Mass Function of Binomial Distribution, $n = 100$



Normal Distribution

- As n increases the binomial distribution approaches the Gaussian or normal distribution

$$pdf(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2 / 4\sigma^2}$$

- 68% of data is within 1 stdev
- 86% of data is within 2 stdev
- 99% of data is within 3 stdev
- Central Limit theorem:
 - Summation of distributions tends toward normal!!

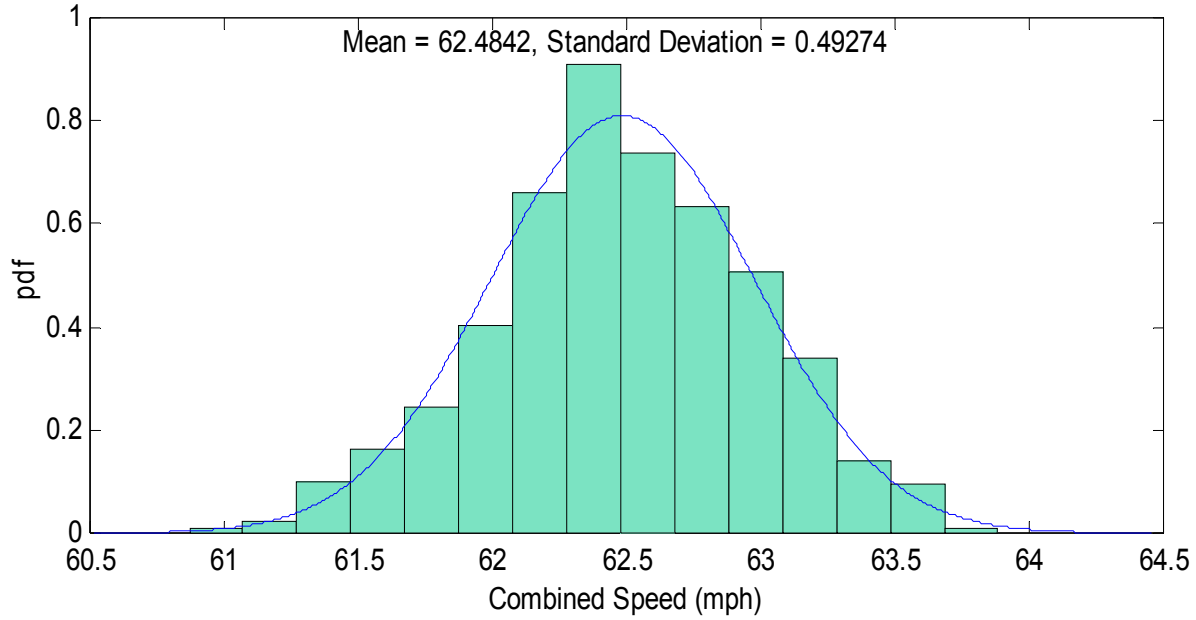
Other Distributions

- Lognormal
- Raleigh
- Uniform
- Triangular
- Poisson's
- Weibul

Simulations

- Random number generators are the core of simulation.
- Coin and die tossing is a form of random number generation
- Computers can do many iterations quickly.
- Example for traffic crash reconstruction
 - Combined Speed

Estimated Probability Density Using Monte Carlo Simulation, M = 1000



Estimated Cumulative Distribution Function Using Monte Carlo, M = 1000

