An Introduction To Rotational Mechanics

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Introduction

- In investigating collisions, situations are encountered where one or more of the vehicles involved experience some degree of spin or rotation.
- The rotational motion may be present along with translational motion.
- Proper interpretation and analysis of the rotational motion may present a challenge for investigators.

Objectives

- The concepts of rotational mechanics will be presented.
- Application will be discussed.
- Case examples will be presented to exemplify the use of rotational mechanics in crash analyses.

DEFINITIONS

Definitions

- **Angular Displacement**
  - Denoted by *2* (theta) and is measured in radians.
  - Angular displacement is analogous to translational displacement measured in feet or meters.

Definitions (cont.)

- **Angular Displacement** (cont.)
  - Formulas:

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\theta = \left(\frac{\omega_o + \omega_f}{2}\right)t$$

Translational equivalent:

$$d = v_o t + \frac{1}{2} a t^2$$

$$d = \left(\frac{v_o + v_f}{2}\right)t$$

Definitions (cont.)

- **Angular Velocity**
  - Change in angular displacement with respect to a change in time.
  - Denoted by *T* (omega) and is measured in radians per second.
  - Angular velocity is analogous to translational velocity measured in ft/sec or m/sec.

Definitions (cont.)

- **Angular Velocity** (cont.)
  - Formulas:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega_f = \omega_o + \alpha t$$

$$\omega_f^2 = \omega_o^2 + 2\alpha\theta$$

Translational equivalent:

$$v = \frac{\Delta d}{\Delta t}$$

$$v_f = v_o + at$$

$$v_f^2 = v_o^2 + 2ad$$

Definitions (cont.)

- **Angular Acceleration**
  - Change in angular velocity with respect to a change in time.
  - Denoted by ″ (alpha) and is measured in radians per second^2.
  - Angular acceleration is analogous to translational acceleration measured in ft/sec^2 or m/sec^2.

Definitions (cont.)

- **Angular Acceleration** (cont.)
  - Formula:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Translational equivalent:

$$a = \frac{\Delta v}{\Delta t}$$

Definitions (cont.)

- **Moment of Inertia**
  - A measure of the resistance a body has to angular acceleration.
  - A function of both the mass and shape of the body.
  - Denoted by *I* (uppercase i) and is measured in slug-ft^2 or lb-ft-sec^2 in the English system and kg-m^2 in the metric system.
  - Moment of inertia is analogous to mass, which is a measure of a body's resistance to translational acceleration.

Definitions (cont.)

- **Moment of Inertia (cont.)**
  - Formula:

$$I = \sum Mr^2$$

Definitions (cont.)

- **Radius of Gyration**
  - A single radius which replaces all the radii of each individual particle of an object and results in the same moment of inertia.
  - Defined by the letter *k*.

Definitions (cont.)

- **Moment of Inertia using Radius of Gyration**
  - Formula:

$$I = Mk^2$$

or

$$k^2 = \frac{I}{M}$$

Definitions (cont.)

- **Torque**
  - Torque is produced by a force multiplied by the length of the lever arm, or moment arm, against which the force is acting.
  - Denoted by *J* (tau) and is measured in ft-lbs or N-m.
  - Torque is analogous to force.

Definitions (cont.)

- **Torque** (cont.)
  - Formula:

$$\tau = F \times L$$

Definitions (cont.)

- **Newton's 2nd Law for Rotation**
  - *Torque = Moment of Inertia X Angular Acceleration*

$$\tau = I\alpha$$

Translational equivalent:

$$F = Ma$$

Definitions (cont.)

- **Angular Impulse**
  - *Impulse = Torque X Time*

$$I = \tau \Delta t$$

Translational equivalent:

$$I = F \Delta t$$

Definitions (cont.)

- **Work done by Rotation**
  - *Work = Torque X Angular Displacement*
  - Measured in ft-lbs or joules.

Definitions (cont.)

- **Work done by Rotation** (cont.)
  - Formula:

$$W = \tau\,\theta$$

Translational equivalent:

$$W = F\,d$$

Definitions (cont.)

- **Kinetic Energy due to Rotation**
  - Energy due to the rotational motion of the object.
  - Measured in ft-lbs or joules.

Definitions (cont.)

- **Kinetic Energy due to Rotation** (cont.)
  - Formula:

$$Ke = \frac{1}{2} I \omega^2$$

Translational equivalent:

$$Ke = \frac{1}{2} M v^2$$

Definitions (cont.)

- **Work-Energy Theorem**
  - The work performed in moving an object is equal to the change in kinetic energy of the object.
  - Measured in ft-lbs or joules.

Definitions (cont.)

- **Work-Energy Theorem** (cont.)
  - Formula:

$$\tau\,\theta = \frac{1}{2} I\omega^2$$

Translational equivalent:

$$F\,d = \frac{1}{2} M v^2$$

Definitions (cont.)

- **Parallel Axis Theorem**

  - The relationship between the rotational inertia of a body about any axis and its rotational inertia about a different, parallel axis, separate from the first.

Definitions (cont.)

- **Parallel Axis Theorem** (cont.)
  - Formula:

$$I = I_G + M h^2$$

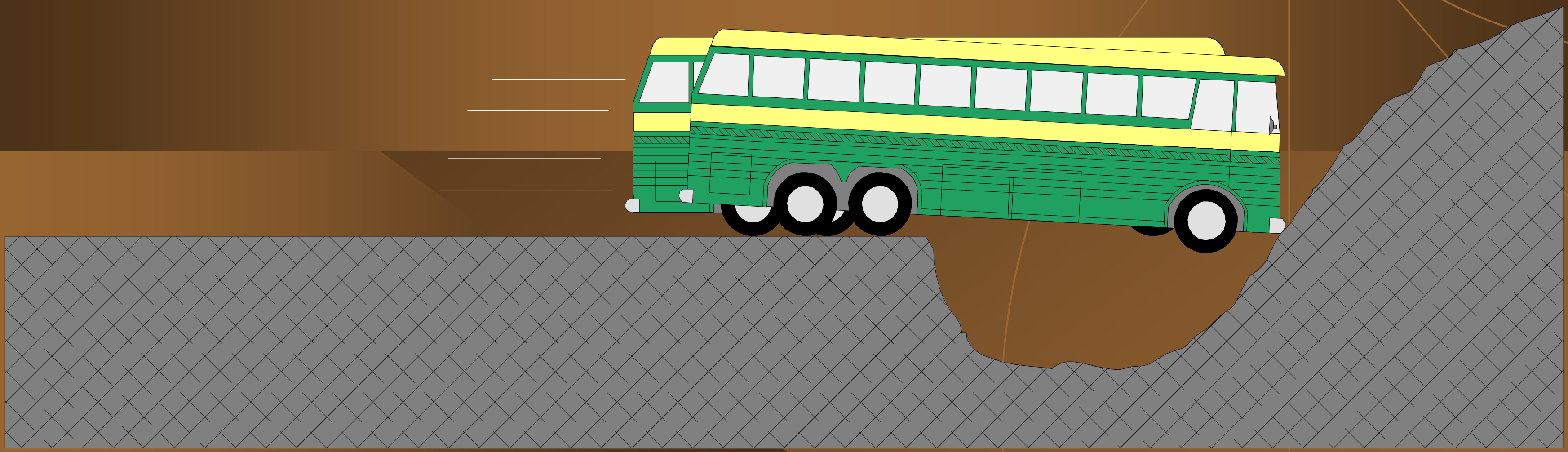
Where I_G = known moment of inertia

M = total mass of the object

h = perpendicular distance between the axes

Case Study #1

Case Study #1



- A bus drove off a level surface and struck the embankment on the other side.
- The bus did not totally leave the surface, the rear axles remained on the surface.
- The bus was wedged against the other side. This was first touch as well as final position.

Case Study #1

- Issue
  - What was the speed of the bus as it left the surface?

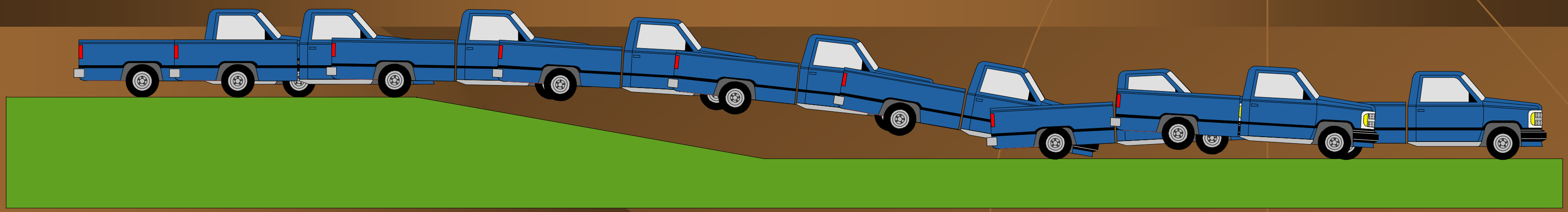
Case Study #1

- Data

w = 29,465 lbs	Weight of bus
$W.B.$ = 285 in.	Wheelbase
CM = 194.9 in.	Behind the front axle
2 = 3^{o} = 0.052 rad	Angular displacement
d = 21.83 ft.	Distance front axle traveled after leaving surface
$I_p = I_G$ = 167,750 slug-ft^2	Pitch moment of inertia

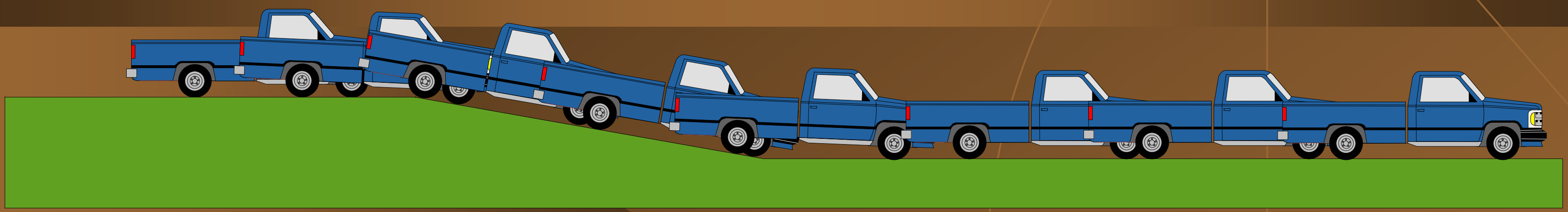
Case Study #2

Case Study #2



- A truck drove off a level surface, went airborne, and landed on a lower surface.
- The truck traveled 31 feet horizontally and fell 6 feet while airborne.
- The calculated speed at take-off was 34 mph.

Case Study #2



- Opposing expert stated the truck did not go airborne but merely drove down the 10^{o} slope to the lower surface.

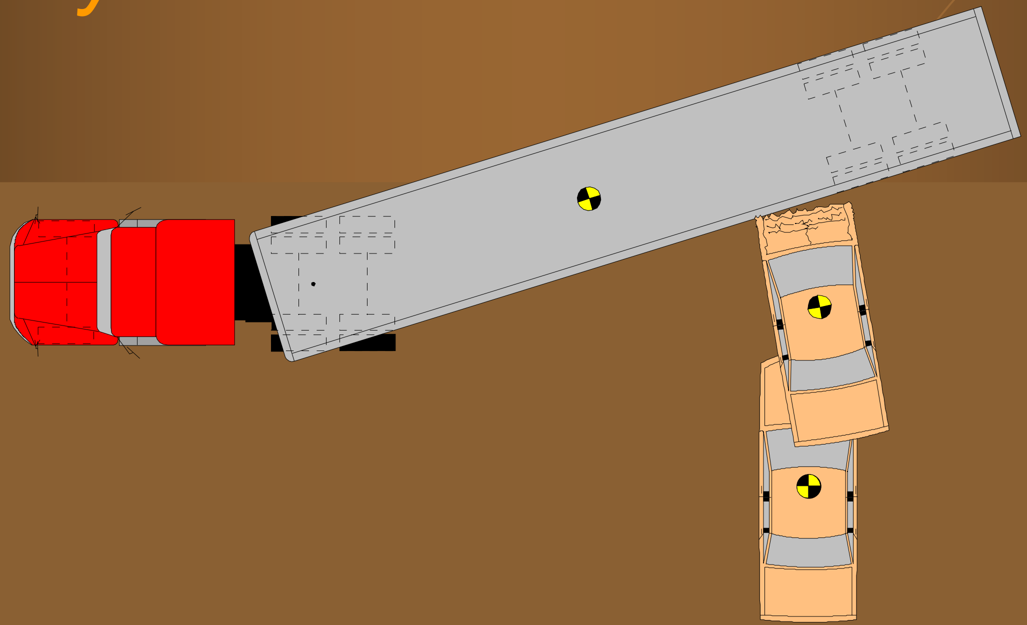
- Issue
  - At what speed will the truck go airborne?

Case Study #2

- Data
  - d = 31 ft. Horizontal airborne distance
  - h = 6 ft. Height fallen while airborne
  - w = 4294 lbs Weight of pick-up truck
  - $W.B.$ = 11 ft. Wheelbase
  - CM = 6.49 ft. In front of the rear axle
  - 2 = 10^{o} = 0.17 rad Angular displacement
  - $I_p = I_G$ = 3102.06 lb-ft-sec^2 Pitch moment of inertia

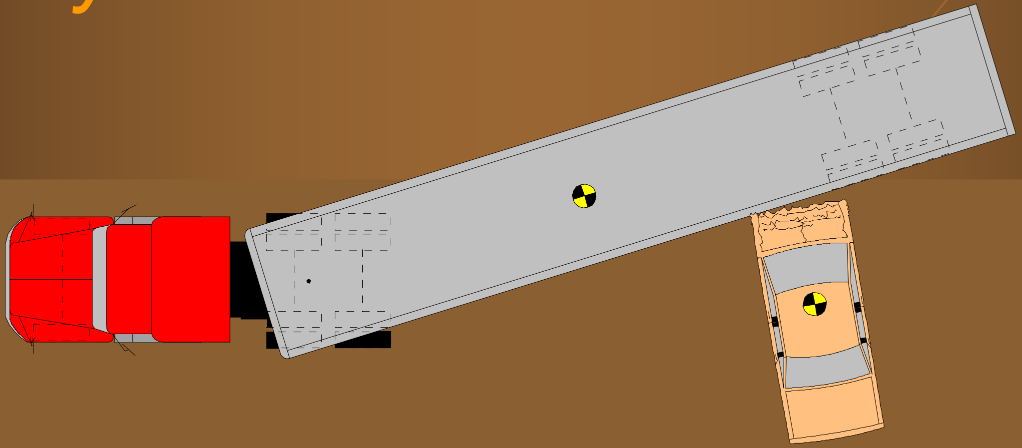
Case Study #3

Case Study #3



- A truck tractor-flatbed semitrailer was parked.
- A car, weighing 4000 lbs., struck the semitrailer in the left side.
- The force of the collision was strong enough to move the semitailer and cause it to pivot about the kingpin.

Case Study #3



- Issue
  - What was the speed of the car at impact?

Case Study #3

- Data

w_1 = 36,000 lbs	Weight of flatbed semitrailer
w_2 = 4000 lbs	Weight of car
w_R = 12,000 lbs	Weight on rear tandems
d_1 = 18 ft.	Distance from kingpin to CM
d_2 = 31 ft.	Distance from kingpin to impact of car
d_3 = 36.83 ft.	Distance from kingpin to center of tandems
2 = 0.3 rad	Angular displacement of rear of semitrailer
$I_y = I_G$ = 170,000 lb-ft-sec^2	Yaw moment of inertia
mu = .60	Coefficient of friction

Further Considerations

Further Considerations

- **How are moments of inertia of vehicles determined?**
  - Tables and documentation
  - Calculations

Further Considerations (cont.)

Determining Moments of Inertia

- The easiest method for obtaining the moments of inertia of cars and light trucks is to use a published database.
- Computer programs such as "Expert AutoStats®" provide data for the pitch, yaw and roll moments of inertia for a wide range of vehicles.

Determining Moments of Inertia (cont.)

- The moments of inertia can also be determined through the use of published regression equations.
- The May/June, 1989 issue of the "Accident Reconstruction Journal", Volume 1, #3, contains an article, "*Vehicle Inertial Parameters*" by Garrott, W.R., et al.
- This article describes the physical measurement methodology used to determine inertial parameters for cars and light trucks.
- The following regression equations were developed and presented in the article.

Determining Moments of Inertia (cont.)

- **Regression Equations for Cars**
  - Yaw Moment = (1.03 x *Weight)* – 1206
  - Pitch Moment = (0.99 x *Weight)* – 1149
  - Roll Moment = (0.18 x *Weight)* – 150

Determining Moments of Inertia (cont.)

- **Regression Equations for Light Trucks**
  - Yaw Moment = (1.03 x *Weight*) – 1343
  - Pitch Moment = (1.12 x *Weight*) – 1657
  - Roll Moment = (0.22 x *Weight*) – 235

Determining Moments of Inertia (cont.)

- **Commercial Vehicle Moments of Inertia**
  - The following information is from DOT HS 807 125, "*A Factbook of the Mechanical Properties of the Components for Single Unit and Articulated Heavy Trucks*", December 1986. Fancher, P.S., et al.
  - The study was conducted by the University of Michigan Transportation Research Institute.

Determining Moments of Inertia (cont.)

- **Commercial Vehicle Moments of Inertia** (cont)
  - Moments of Inertia will help us determine how commercial vehicles may react in certain driving conditions.
  - Method for calculating various moments of inertia are found in the reference.
  - Yaw and Pitch Moments are very similar for commercial vehicles.
  - All references are to Moments of Inertia about an axis going through the CM.

Determining Moments of Inertia (cont.)

- **Commercial Vehicle Moments of Inertia** (cont)
  - The ***Yaw Moment*** will assist in the study of response times, rearward amplification, transient braking, and response to disturbances such as wind loading.
  - The ***Pitch Moment*** is useful in studying transient braking response, such as ABS or driver initiated intermittent braking. The pitch moment may also assist us with certain speed calculations.
  - The ***Roll Moment*** is smaller than the other two. It is used when studying dynamic rollover situations.

Determining Moments of Inertia (cont.)

- **Truck & Tractor Yaw and Pitch Moments**

Ford 9000 Tractor:	26,560	lb-ft-sec^2
GMC Astro 95 Tractor:	20,123	lb-ft-sec^2
Ford 800 Tractor:	15,946	lb-ft-sec^2
IH Tractor:	14,730	lb-ft-sec^2
GMC Astro 95 Dump, empty:	14,713	lb-ft-sec^2
GMC Tractor:	11,546	lb-ft-sec^2
GMC 6500 V-8 Dump, empty:	10,970	lb-ft-sec^2

Further Considerations (cont.)

Determining Moments of Inertia (cont.)

- **Empty Semitrailer Pitch and Yaw Moments**

48’, Tandem, 40’ WB, 13,800 lb:	110,739	lb-ft-sec^2
45’, Tandem, 37’ WB, 13,043 lb:	91,157	lb-ft-sec^2
42’, Tandem, 36’ WB, 12,286 lb:	78,751	lb-ft-sec^2
28’, Single, 22.8’ WB, 6,753 lb:	39,627	lb-ft-sec^2
27’, Single, 21’ WB, 6,500 lb:	34,600	lb-ft-sec^2

Determining Moments of Inertia (cont.)

- **Loaded Semitrailer Pitch and Yaw Moments**
- Homogeneous cargo @ 14 lb/ft^3
  - 48', Tandem, 40' WB, 60,500 lb: 403,539 $lb\text{-}ft\text{-}sec^2$
  - 45', Tandem, 37' WB, 56,843 lb: 333,878 $lb\text{-}ft\text{-}sec^2$
  - 42', Tandem, 36' WB, 53,086 lb: 278,217 $lb\text{-}ft\text{-}sec^2$
  - 28', Single, 22.8' WB, 33,952 lb: 100,921 $lb\text{-}ft\text{-}sec^2$
  - 27', Single, 21' WB, 32,750 lb: 89,848 $lb\text{-}ft\text{-}sec^2$

Further Considerations

- **How are eccentric collisions, where one vehicle is “swept” aside, handled?**
  - Utilize the “*Effective Mass Ratio*”
  - Calculate) *V*
  - Perform traditional conservation of linear momentum computation.

Utilizing Effective Mass Ratio (cont.)

- Consider the collision where a stationary vehicle is eccentrically struck in the side, towards the front, by another vehicle.
- The collision pivots the stationary vehicle out of the way as the bullet vehicle continues to a stop.
- The stationary vehicle does not have a great deal of post collision translational motion.

Utilizing Effective Mass Ratio (cont.)

- In this collision the target vehicle was not entirely displaced by the collision.
- The full effect of the mass of the target vehicle was not felt by the bullet vehicle.
- A fair representation of the departure speed of the target vehicle might not be obtained by traditional methods.
- Therefore, the *effective mass ratio* of each vehicle should be calculated and used to determine delta-Vs.
- Then a traditional COLM calculation can be performed.

Utilizing Effective Mass Ratio (cont.)

- **Effective Mass Ratio**
  - Formula:

$$\gamma = \frac{k^2}{h^2 + k^2}$$

Where k = radius of gyration

h = lever arm on which the collision force acts

Utilizing Effective Mass Ratio (cont.)

- Use the calculated effective mass ratios of both vehicles to calculate the delta-V of the target vehicle.

Further Considerations (cont.)

Utilizing Effective Mass Ratio (cont.)

- Delta-V formula:

$$\Delta v_2 = \sqrt{\frac{2g\gamma_2\left(E_1 + E_2\right)}{w_2\left(1 + \dfrac{\gamma_2 w_2}{\gamma_1 w_1}\right)}}$$

Where

$(_1$, $(_2$ = respective effective mass ratios of bullet and target vehicles

w_1, w_2 = respective weights of bullet and target vehicles

E_1, E_2 = respective amount of energy needed to damage the bullet and target vehicles

g = gravity

Utilizing Effective Mass Ratio *(cont.)*

- Having calculated delta-V, this is also the departure speed of the target vehicle since it was stationary.
- The impact speed of the bullet vehicle can now be calculated using the traditional COLM equation.

$$v_1 = v_3 + \frac{w_2 v_4}{w_1} - \frac{w_2 v_2}{w_1}$$

CONCLUSIONS

Conclusions

- A large number of situations can be solved using standard rectilinear techniques and equations.
- However there are those situations which present rotational issues which must be considered.
- Understanding and utilizing proper theories and formulas from rotational mechanics allows solutions to these situations to be found.
- As reconstructionists, we must continually strive to add to our toolbox of knowledge to be able to do the best job we can.