Vehicle Dynamics - Adjusting Drag Factors for Special Circumstances

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Prepared for use in IPTM Training Programs

The Problem

- Vehicles may skid on a road or surface that changes slope as the skid progresses.
- Even if the surface material is the same, the drag factor will change as the slope changes.
- We will have to account for this.

The Problem

- In the same way, a spinning vehicle will have a constantly changing drag factor if some wheels are not braking.
- The drag factor will change as a function of the spin angle.
- We will see how we may account for this constantly changing drag factor

The Problem

- In our previous training, we have learned the *Combined Speed Equation* will help us to compute the speeds of vehicles crossing different surfaces.
- This same equation will help us compute speed *anytime* we are confronted with changing drag factors!

The Combined Speed Equation

$$S = \sqrt{S_1^2 + S_2^2 + S_3^2 + \cdots + S_n^2}$$

$$S = \sqrt{30(f_1 d_1 + f_2 d_2 + f_3 d_3 + \cdots + f_n d_n) + S_i^2}$$

- Either of these equations works the same way: Summing up *Kinetic Energy Equivalent Speeds (KEES)*

What Are We Going to Do?

- We know speed is a function of drag factor and distance.
- We will discuss how to calculate changing drag factors.
- We will look at correcting drag factor for changing slopes (hills).
- We will address post-impact spins by developing a simple spin model using a modified drag factor equation.

Hill Adjustment

Hill Drag Factor Adjustment

- Adjusting the drag factor for road alignments or conditions is a normal crash reconstruction procedure.
- The basic drag factor equations:

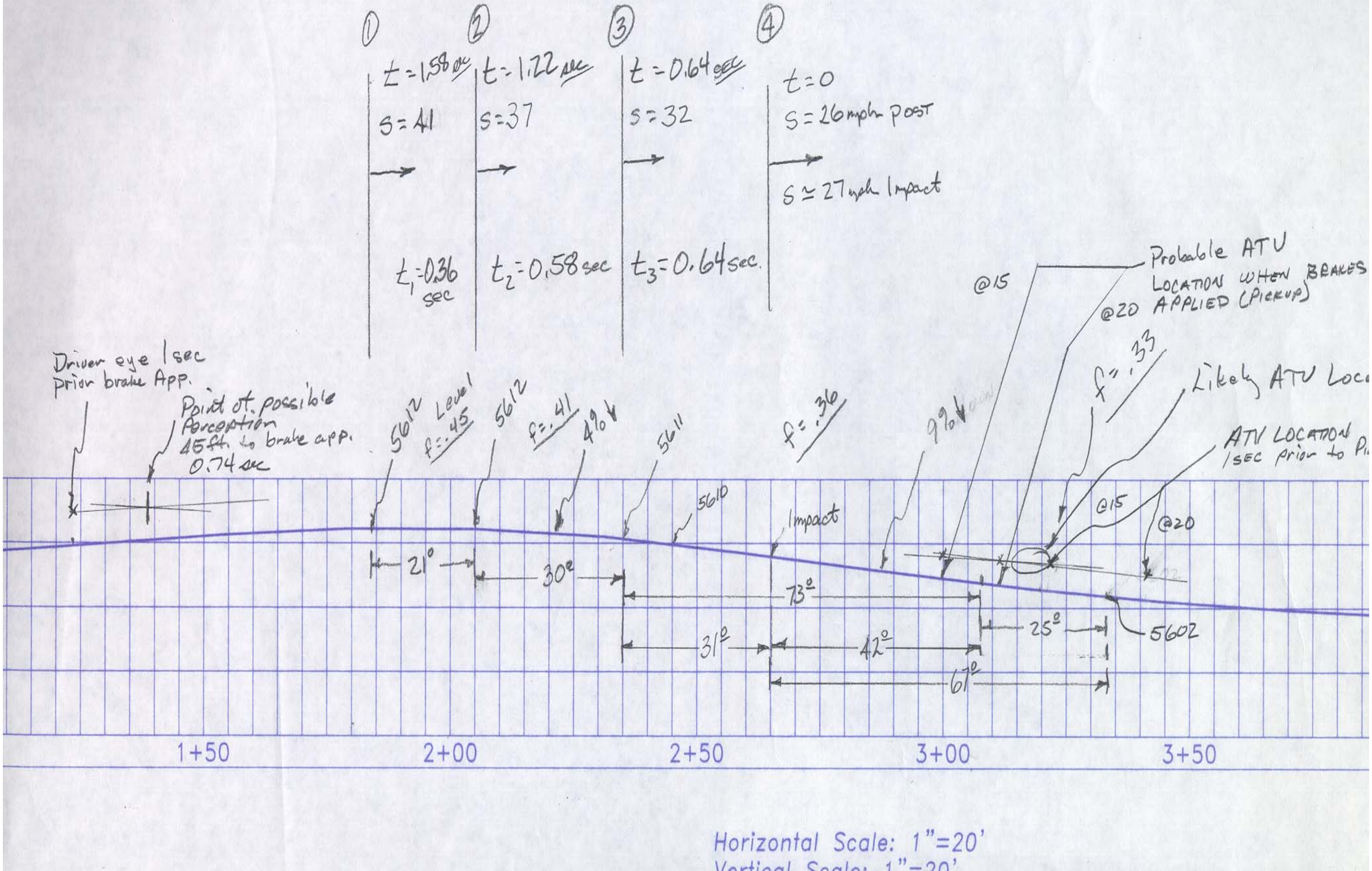
$$f = \mu n \cos\theta + \sin\theta$$

$$f = \frac{\mu n \pm m}{\sqrt{1 + m^2}}$$

$$f = \mu n \pm m$$

Hill Drag Factor Adjustment, cont'd.

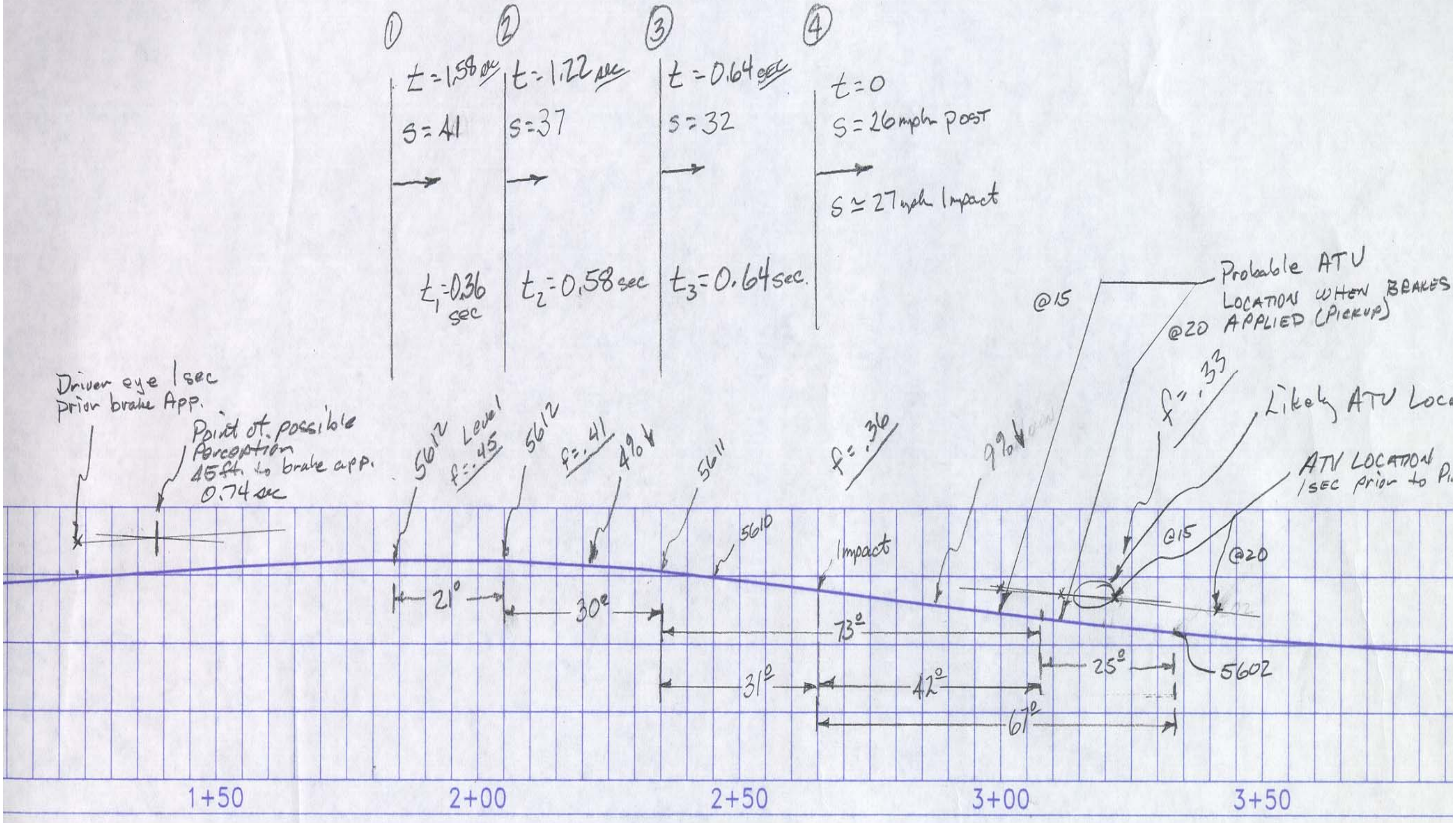
- In these equations:
  - μ = surface friction coefficient
  - θ = road slope in degrees
  - n = percentage of braking
  - m = slope of road in percent (decimal)
- Any of these equations work for a constant road slope.
- What if the slope of the road varies as the skid progresses?



1
t = 1.58 sec
S = 41
t1 = 0.36 sec
2
t = 1.22 sec
S = 37
t2 = 0.58 sec
3
t = 0.64 sec
S = 32
t3 = 0.64 sec.
4
t = 0
S = 26 mph POST
S ≃ 27 mph Impact
Probable ATU LOCATION WHEN BRAKES APPLIED (Pickup)
@15
@20
Driver eye 1 sec prior brake App.
Point of possible Perception 45 ft. to brake app. 0.74 sec
5612
Level
f = .45
5612
f = .41
4% ↓
5611
5610
f = .36
Impact
9% ↓
f = .33
Likely ATV Loc
ATV LOCATION 1 sec prior to Pi
@15
@20
21'
30'
73'
31'
42'
25'
67'
5602
1+50
2+00
2+50
3+00
3+50
Horizontal Scale: 1"=20'
Vertical Scale: 1"=20'

Hill Drag Factor Adjustment, cont'd.

- This scale profile drawing illustrates the skidding path of a vehicle on a county road.
- This vehicle was skidding straight with no appreciable rotation.
- The skid starts on the level surface and then progresses to different downhill slopes.
- We must account for the changing slope if we are to compute the correct speed.



Horizontal Scale: 1"=20'
Vertical Scale: 1"=20'

Hill Drag Factor Adjustment, cont'd.

- In this case, the initial investigators considered only the steepest part of the slope. In addition, the skid mark was measured short by the vehicle wheelbase.
- The speed computed was 34 mph.
- The proper way to handle the problem is to compute drag factors for each section of constant slope.
- This information is then put into the combined speed equation.
- When this is done, the computed speed rises to 41 mph.

Hill Drag Factor Adjustment, cont'd.

- Clearly, there are cases where we need to adjust the drag factor as the slope of the road changes.
- By calculating a new drag factor for each distance increment along the vehicle path, we may then use the *Combined Speed Equation* to compute a more correct speed
- This is a simple form of numerical integration (quadrature).

Drag Factor Adjustments for Spin

Adjustments for Spin

- Vehicles may spin out after a collision or may rotate into a spin after entering a critical speed yaw.
- Sometimes only one or two of the wheels have any effective braking, perhaps due to damage.
- Most times, there is no driver applied braking.
- The drag on the free-rolling tires will be a function of the angle the tires make with respect to the vehicle motion.
- The minimum level drag factor for a free-rolling wheel is about 0.01 to 0.02. Powered wheels are about 0.03 in high gear.
- Let’s look at some definitions:

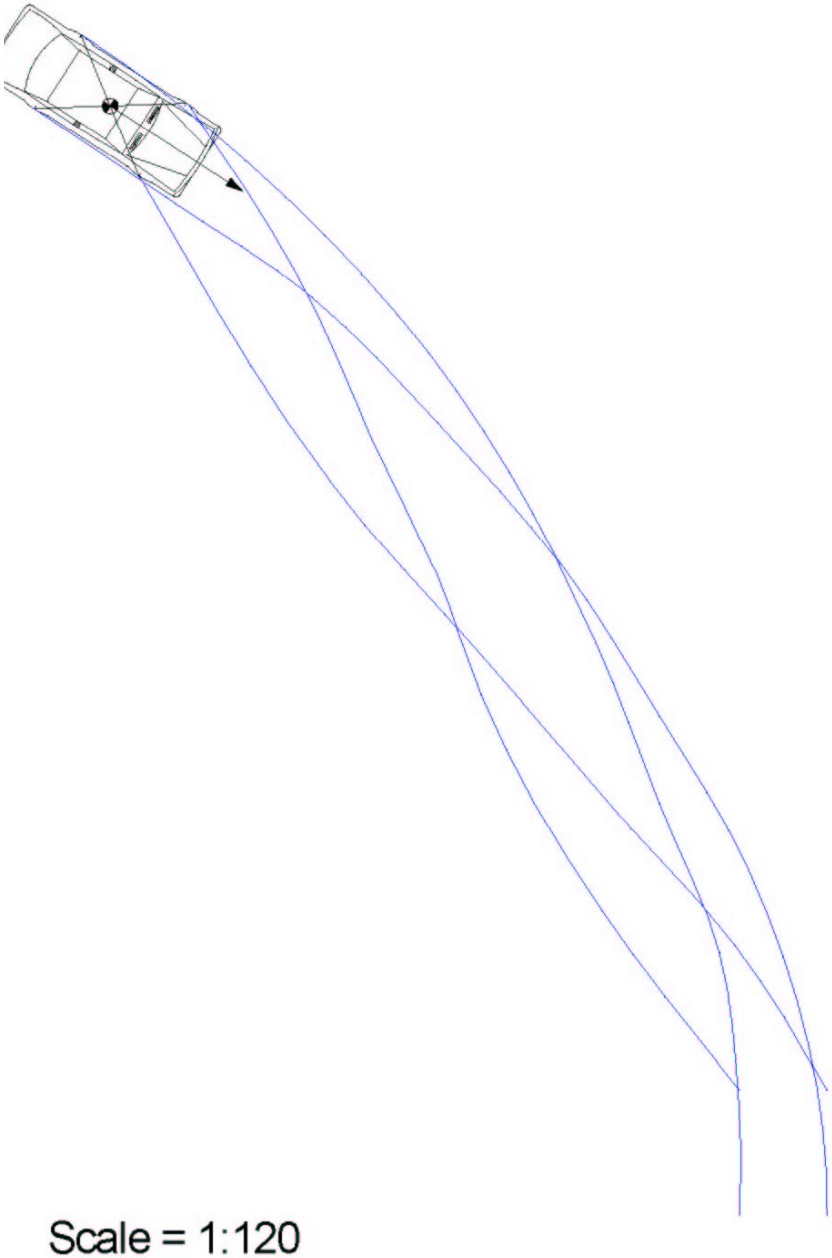
Adjustments for Spin

- The ***heading*** of a vehicle is the direction the headlights are pointing.
- The ***bearing*** of the vehicle is the direction the center of mass is moving…the velocity vector of the vehicle.
- The *difference* between the two is the ***slip angle*** of the vehicle, denoted by α.
- When the vehicle is going straight ahead, its overall drag will be at a minimum.
- We will define the minimum ***level*** surface drag factor as f_r
- When the vehicle is sideways, its drag will be maximum.
- As the vehicle progresses through a spin, its drag factor will be changing because the slip angle is changing.
- We may use the following equation to compute drag factor:

Adjustments for Spin

$$f_{adj} = [f_r + (\mu - f_r)|\sin\alpha|] \pm m$$

- Where:
  - f_r = minimum level road drag seen by the vehicle
  - μ = road or surface friction coefficient
  - α = The slip angle of the vehicle, or the difference between the heading and the bearing of the vehicle.
  - m = the slope of the surface in the direction of the bearing
  - The vertical bars on either side of sin α indicates *absolute value,* which means we use the positive (+) sine value
- Now, let's consider the following problem:



This vehicle has come to rest after impact, leaving the tire marks shown.

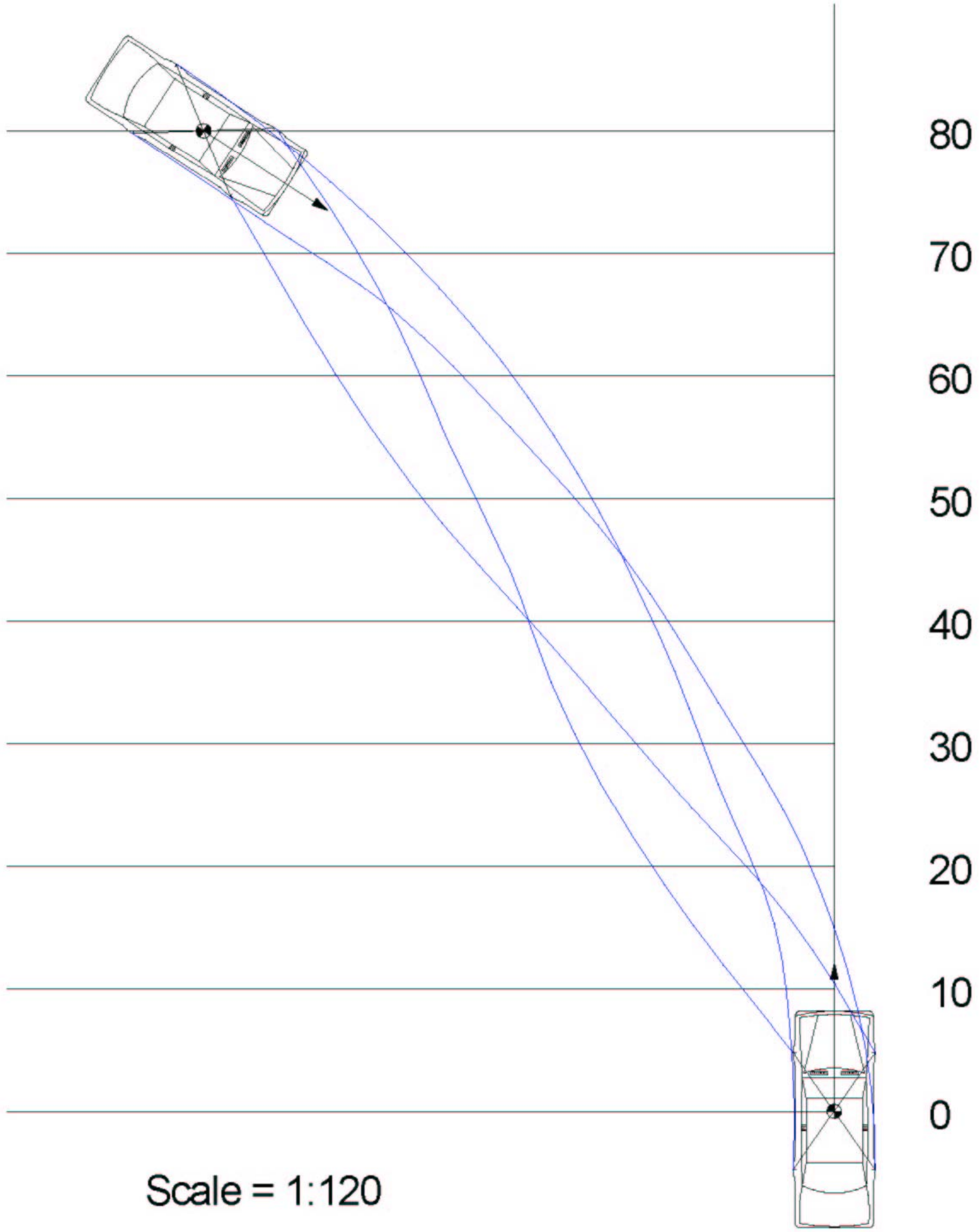
If we confuse these tire marks with critical speed yaw marks, we will calculate an erroneous speed.

The speed we calculate will likely be too high.

The following procedure will help us determine a more accurate speed.

We will calculate drag factors for small distance increments and will use the combined speed equation.

It is important that we look at the condition of each wheel, as some may be retarding more than others. Some similar techniques fail to do this.



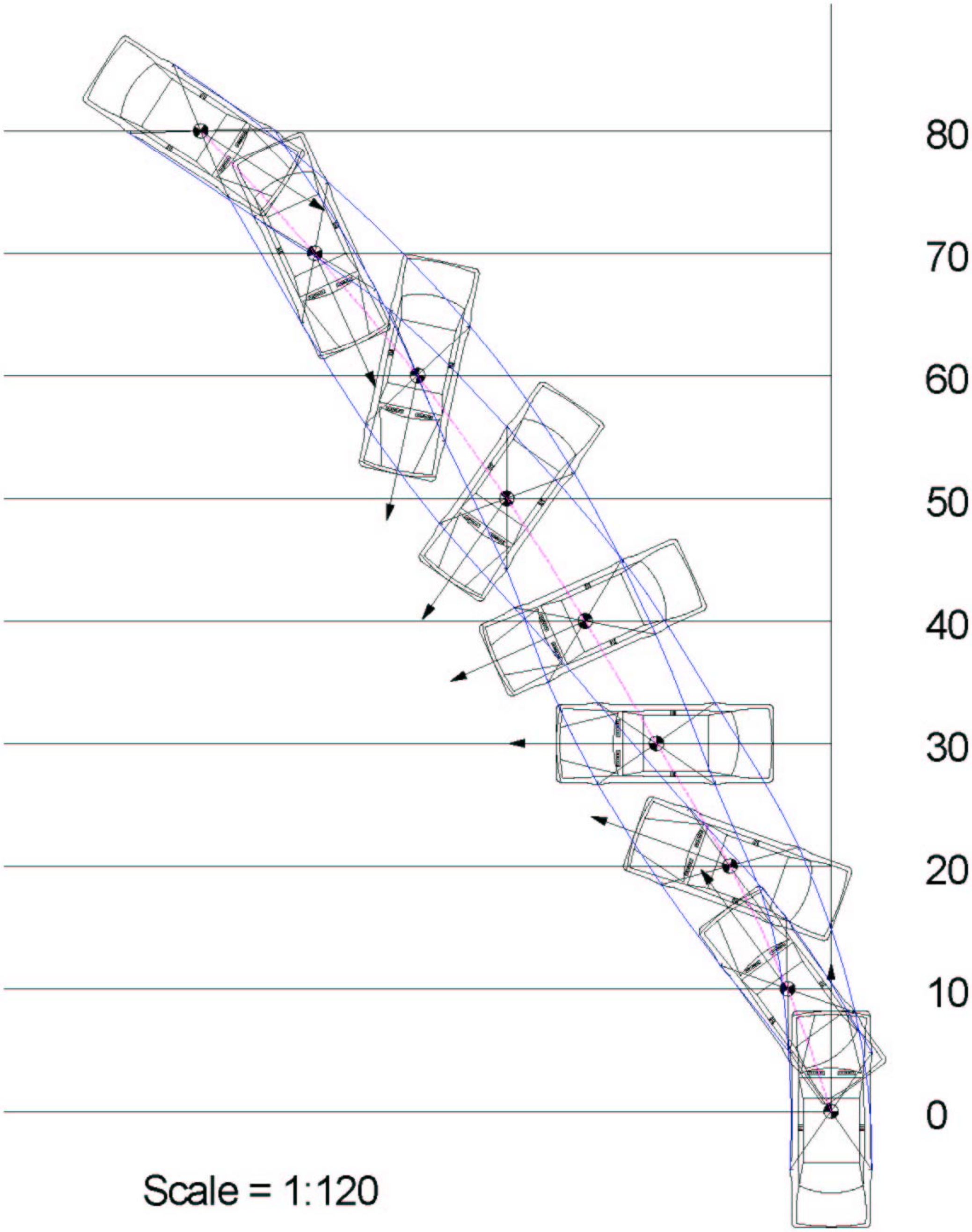
Plot the vehicle back at its spin initiation point.

Draw a station line parallel to the initial velocity vector of the vehicle.

Draw offset lines perpendicular to the station line every 10 or 20 feet.

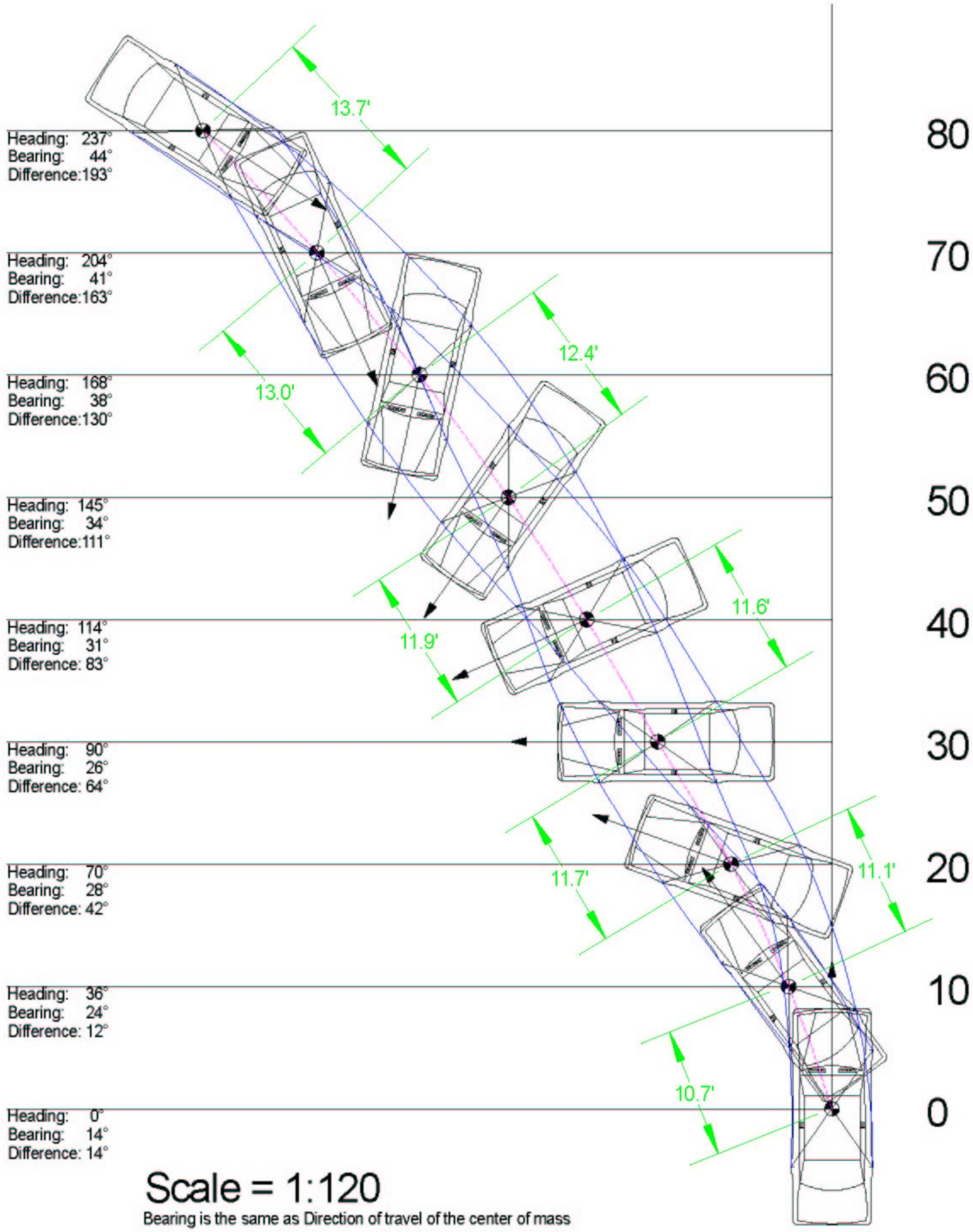
These lines do not always have to be the same distance apart.

The last offset line will be through the center of mass at rest, and may not be the same as the others.



The heading of the vehicle is the direction in which the headlights are pointing.

Plot the vehicle heading by placing the vehicle back on its tire marks, locating the center of mass on the appropriate offset line.



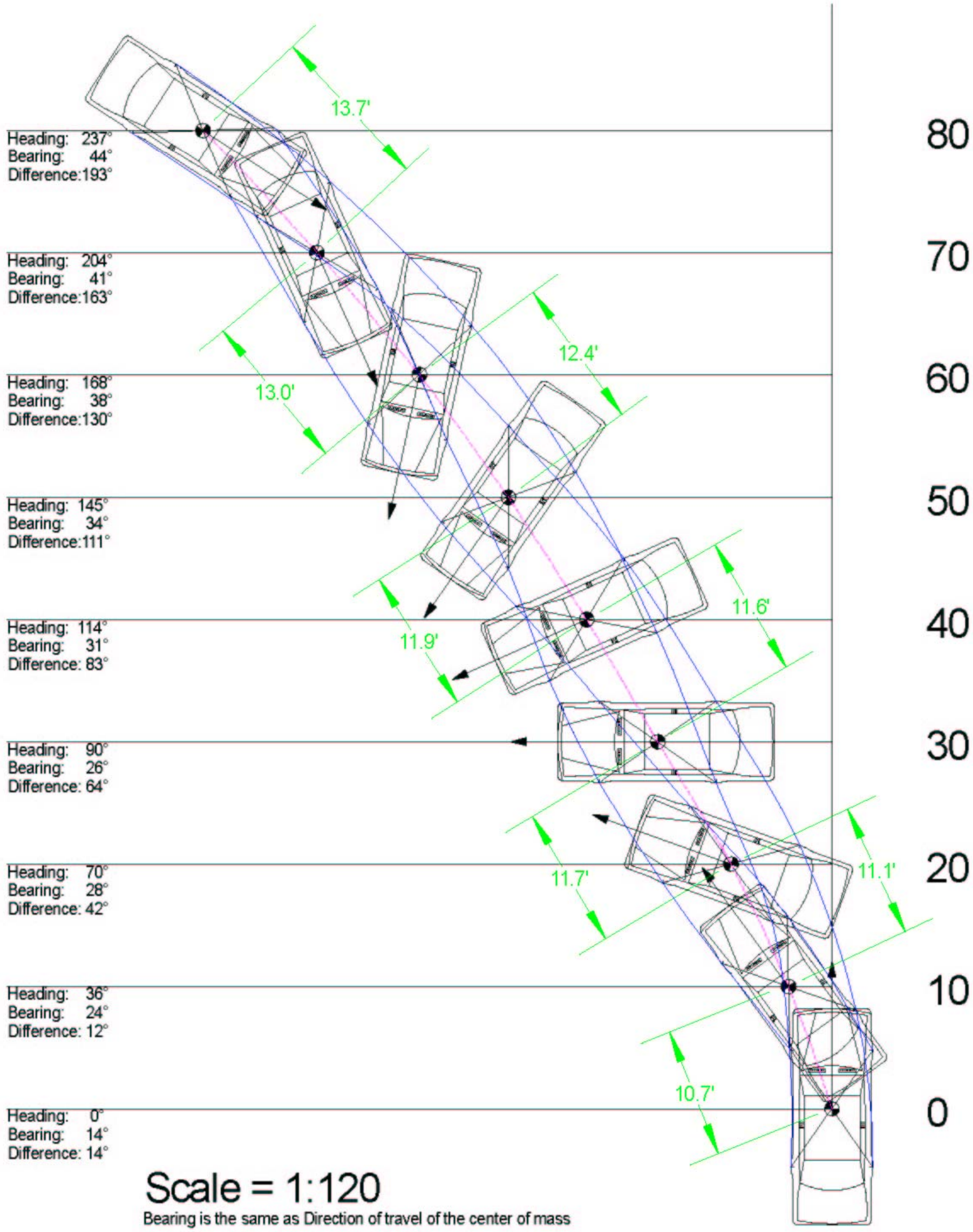
Determine the bearing by plotting the tangent to the vehicle path at each CM location.

The ***slip angle*** will be denoted as **α** (alpha), and is the *difference* between the heading and the bearing.

Use the CAD program to determine the center of mass distance moved between each offset line.

We will need to calculate the *average* slip angle between two offset lines.

$$\alpha_{ave} = \frac{\alpha_i + \alpha_{i+1}}{2}$$



Once we have determined the average slip angle over an interval, we may then calculate a drag factor over that interval.

After we have computed the drag factor over the interval. we may use the distance across the interval to compute the KEES across that interval by using the Basic Speed Equation.

$$S - \sqrt{30df}$$

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Drag Factor Calculation

- In this example, we are considering none of the wheels to be locked due to damage. Engine braking only.
- In addition, we consider the vehicle stopped without rolling out at the end after the spin.
- In most cases with unlocked wheels, the vehicle will roll out for some distance after the initial spin.
- A roll out is defined when the rear tires track with the front tires without skidding.
- If we have a roll out, we treat it as another term in the combined speed equation.
- For our simple model, we may develop the following table:

For f_r=0.06; m=0; μ=0.75 (This simulates partial engine braking only)

Interval	d	α_{ave}	μ	f_r	Sin α	m	f_{adj}	S mph
1	10.7 ft	13°	0.75	0.06	0.225	0	0.22	8.31
2	11.1 ft	27°	0.75	0.06	0.450	0	0.37	11.10
3	11.7 ft	53°	0.75	0.06	0.798	0	0.61	14.63
4	11.8 ft	73.5°	0.75	0.06	0.958	0	0.72	15.98
5	11.9 ft	97°	0.75	0.06	0.992	0	0.74	16.30
6	12.4 ft	120.5°	0.75	0.06	0.861	0	0.65	15.60
7	13.0 ft	146.5°	0.75	0.06	0.552	0	0.44	13.11
8	13.7 ft	178°	0.75	0.06	0.035	0	0.08	5.88
Total	96.1 ft							37.11

Let’s Calculate the First Row

$$\alpha_{ave} = \frac{\alpha_i + \alpha_{i+1}}{2}$$

$$\alpha_{ave} = \frac{14 + 12}{2}$$

$$\alpha_{ave} = 13°$$

Let's Calculate the First Row

$$S = \sqrt{30df}$$

$$S = \sqrt{30(10.7)(0.22)}$$

$$S = 8.31mph$$

Now Combine the Speeds

$$S = \sqrt{S_1^2 + S_2^2 + S_3^2 + S_4^2 + S_5^2 + S_6^2 + S_7^2 + S_8^2}$$

$$S = \sqrt{8.31^2 + 11.10^2 + 14.63^2 + 15.98^2 + 16.30^2 + 15.60^2 + 13.11^2 + 5.88^2}$$

$$S = 37.11 mph$$

What is the Drag Factor Ratio?

- We will define a drag factor ratio with the Greek letter η (eta).
- It is used in a similar manner as n, the percentage of braking.
    - $f = \mu\eta + m$
- This is simply the ratio of the level road computed drag factor for the spin divided by μ. ($\eta = f/\mu$)
- In this case, $\eta = 0.64$

What is the Drag Factor Ratio?

- Essentially, this tells us the spin is 64% as efficient at stopping the vehicle as compared to a straight skid.

Spin Adjustments

- The method allows us to compute a drag factor for each distance increment.
- This allows us to account for variations in braking between the wheels and accounting for the spin angle.
- If one or more wheels are locked, we may handle the problem in a straightforward manner by defining a minimum level road drag factor, f_r
- We account for road slope for each distance increment.
- This method does NOT calculate any differences that result from high rates of spin (angular velocity)
- The computed drag factor is not exact, but will get us a reasonable result, and is more correct than a “rule of thumb”.

Summary

- Drag factor adjustments for constant slopes are straightforward.
- Sometimes the slope is NOT constant, especially on city streets and county roads.
- The changing slope will result in a changing drag factor, which must be accounted for.
- Post-impact spins are not difficult to handle even when there is no braking.

Summary

- Plot the path, compute the slip angle, and account for the grade.
- The minimum level drag factor for any given wheel will be on the order of 0.01 to 0.02. Powered wheels will be about 0.03 in high gear.
- For situations with rollout after the spin, carefully identify the point where the rollout starts. Use the spin analysis to that point, then a rolling drag factor for the remainder. It's all *COMBINED SPEED!*